

حمل الآن

مجاناً وحصرياً

المراجعة رقم (1)

اختبار شهر اكتوبر





Quiz

1

on lesson 1 – unit 1

Total mark

10

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = \hat{i} + \hat{j}$ where F_1 , F_2 are measured in newton then the magnitude of their resultant newton.

(a) $\sqrt{2}$

(b) $\sqrt{5}$

(c) $\sqrt{13}$

(d) 5

- (2) Two forces are equal act at a point and the measure of the angle between them is $\frac{\pi}{3}$ and their resultant is 3 newton , then the magnitude of each is newton.

(a) $\frac{3}{2}$

(b) $\sqrt{3}$

(c) 3

(d) $3\sqrt{3}$

- (3) The resultant of two forces acting at a point is maximum when the included angle between them is equal to

(a) zero

(b) 60°

(c) 120°

(d) 180°

- (4) The magnitude of the resultant of two forces 3 , 5 newton and the measure of their included angle is 60° equals newton.

(a) 2

(b) 6

(c) 7

(d) 8

Second question

3 marks

The magnitude of two forces are F , 4 newton acting at a point , and the measure of the angle between them is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton , find the magnitude of $\vec{F}$ and the angle measure between their resultant and the force $\vec{F}$

Third question

3 marks

The magnitude of two forces are 4 , F newton acting at a point , and the measure of the angle between them is 120° , their resultant is perpendicular on the first force. Find the value of F

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) Two forces of magnitude $3F$ and $2F$ intersecting at a point and their resultant is $5F$, then the measure of the angle between them is

(a) 0° (b) 60° (c) 20° (d) 180°

- (2) By resolving the force $\vec{R}$ into two forces $\vec{F}_1$ and $\vec{F}_2$ making with $\vec{R}$ two angles of measure θ_1 and θ_2 on both sides of $\vec{R}$ respectively, then the magnitude of $\vec{F}_1 = \dots\dots\dots$

(a) $\frac{R \sin \theta_1}{\sin (\theta_1 + \theta_2)}$ (b) $\frac{R \sin \theta_2}{\sin (\theta_1 - \theta_2)}$ (c) $\frac{R \sin \theta_2}{\sin (\theta_1 + \theta_2)}$ (d) $\frac{R \sin (\theta_1 + \theta_2)}{\sin \theta_2}$

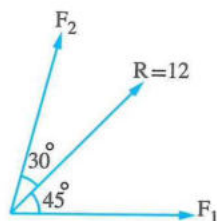
- (3) Two forces of equal magnitudes, inclosing between them an angle of measure 90° . If the magnitude of their resultant is 8 N , then the value of each force measured in newton is

(a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8

- (4) In the given figure :

$F_1 = \dots\dots\dots$

(a) $12 \cos 75^\circ$ (b) $12 \cos 45^\circ$
(c) $6 \sec 45^\circ$ (d) $6 \csc 75^\circ$



Second question

3 marks

Two forces of magnitudes 4 N and $F\text{ N}$ act at a point and the measure of their included angle is 135° . Given that their resultant makes angle 45° with the force F , find F and the magnitude of their resultant.

Third question

3 marks

Resolve a force 100 N in two directions the first inclines by 60° to the force and the other by 30° in the other side of the given force.

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) A body of weight (W) is placed on an inclined plane makes angle of measure θ to the horizontal if the weight is resolved into two perpendicular directions , then the magnitude of the component of its weight in direction of line of greatest slope equals
- (a) $W \cos \theta$ (b) $W \sin \theta$ (c) $W \tan \theta$ (d) W
- (2) Two perpendicular forces of magnitude 12 newton , 5 newton act at a point , then the magnitude of their resultant = newton.
- (a) 17 (b) 7 (c) 13 (d) 14
- (3) Given : $\vec{F}_1 = 3 \hat{i} - 2 \hat{j}$, $\vec{F}_2 = a \hat{i} - \hat{j}$, $\vec{F}_3 = 4 \hat{i} - b \hat{j}$ and their resultant $\vec{R} = 6 \hat{i} - 4 \hat{j}$, then $a + b = \dots\dots\dots$
- (a) 2 (b) -2 (c) zero (d) -1
- (4) Given : $\vec{F}_1 = 5 \hat{i}$, $\vec{F}_2 = 7 \hat{i} - 5 \hat{j}$, $\vec{R}$ is their resultant , then $\|\vec{R}\| = \dots\dots\dots$
- (a) $\sqrt{5} + \sqrt{74}$ (b) 49 (c) 13 (d) $\sqrt{12} - \sqrt{5}$

Second question

3 marks

Three coplanar forces of magnitudes 85 , 75 , $50\sqrt{2}$ kg.wt. act at a point , the first acts towards East , the second towards 30° West of the North and the third towards West South. Find the magnitude of their resultant.

Third question

3 marks

Two forces act at a point , the maximum value of their resultant is 32 kg.wt. and the minimum value of their resultant is 12 kg.wt. Find the magnitude of each force , then find the magnitude of their resultant when the angle between the two forces = 60°

Answer the following questions :

First question

4 marks

1 mark for each item

Choose the correct answer from those given :

- (1) Three equal forces in magnitudes act at a point and the forces are in equilibrium, then the measure of the angle between any two forces =
- (a) 60° (b) 90° (c) 120° (d) 150°
- (2) The maximum and minimum value respectively of the resultant of the two forces of magnitudes 8, 13 newton are newton.
- (a) 13, 8 (b) 13, 5 (c) 21, 8 (d) 21, 5
- (3) Two forces act at a point of magnitudes 15, 9 newton and the measure of the angle between them is 60° then the magnitude of their resultant (R) equals newton.
- (a) 6 (b) 21 (c) 24 (d) 15
- (4) Two forces of equal magnitudes, the magnitude of their resultant is $2\sqrt{3}$ newton and the measure of the angle between them is $\frac{\pi}{3}$, then the magnitude of each newton.
- (a) 2 (b) $\sqrt{2}$ (c) 1 (d) $2\sqrt{2}$

Second question

3 marks

A body of weight 300 gm.wt. is placed on a smooth plane inclined to the horizontal with an angle whose tangent equals $\frac{1}{\sqrt{3}}$. The body is prevented from sliding by a force makes with the line of the greatest slope an angle of measure 30° upwards.

Find the magnitude of the force and the reaction of the plane.

Third question

3 marks

If $\vec{F}_1 = 5\vec{i} + 3\vec{j}$, $\vec{F}_2 = a\vec{i} + 6\vec{j}$, $\vec{F}_3 = -14\vec{i} + b\vec{j}$ are three coplanar forces meeting at a point and their resultant is $\vec{R} = (10\sqrt{2}, \frac{3\pi}{4})$, then find the values of a and b

Answer the following questions :

First question

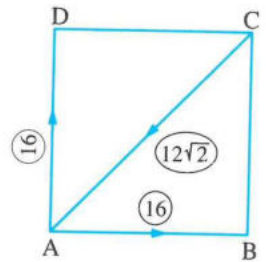
2 marks

The magnitudes of two forces F , $\sqrt{2} F$ newton act at a point and their resultant is perpendicular to the first force. Find the angle between the two forces and prove that the magnitude of their resultant equals F

Second question

2 marks

The opposite figure represents the forces of magnitudes 16 , 16 , $12\sqrt{2}$ newton which act in the square ABCD in the directions $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{CA}$ respectively. Find the magnitude and direction of their resultant.



Third question

4 marks

A smooth sphere of radius length 30 cm. and of weight 10 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 30 cm. , one of its ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the tangency point of the sphere and the wall.

Find the magnitude of the tension in the string and the magnitude of the reaction of the wall.

Fourth question

2 marks

Three coplanar forces of magnitudes 5 , 10 , $4\sqrt{7}$ newton act at a point , the measure of the angle between the first two forces equals 60° , find the greatest and the smallest magnitude of their resultant.

Test

1

Total mark

10

1 Choose the correct answer from the given ones :

(6 marks)

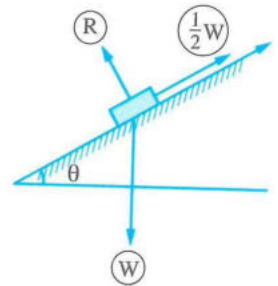
(1) Two forces of magnitudes 8 and 16 kg.wt. and the measure of their included angle is 120° . If these two forces act at a body, then the direction of motion of the body makes an angle of measure with the smaller force.

- (a) 30° (b) 90° (c) 60° (d) 45°

(2) In the opposite figure :

If the body is in equilibrium under action of the shown forces, then $m(\angle \theta) = \dots\dots\dots$

- (a) 30° (b) 60°
(c) 45° (d) 15°

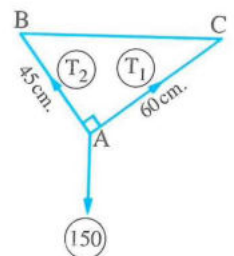


(3) F N. and K N. are the magnitudes of two forces where $F > K$. If the smallest and the greatest value of their resultant are 5, 9 newton respectively, then $5F - 2K = \dots\dots\dots$ N.

- (a) 53 (b) 31 (c) 49 (d) 4

(4) In the opposite figure :

A body of weight 150 gm. wt. is in equilibrium by suspended by two perpendicular strings their lengths are 60 cm., 45 cm. and the other ends are fixed at C and B on the same horizontal line, then $T_2 - T_1 = \dots\dots\dots$ gm.wt.



- (a) 120 (b) 90 (c) 60 (d) 30

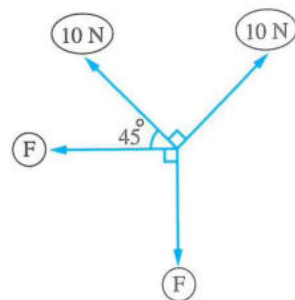
(5) Forces of magnitudes 8, $4\sqrt{3}$, $6\sqrt{3}$, 14 newton act at a point. The measure of the angle between the first and second force is 30° and between the second and third is 120° and between the third and fourth is 90° , then the magnitude of their resultant =

- (a) 4 (b) 6 (c) 8 (d) 7

(6) In the opposite figure :

The condition of equilibrium of the given forces is

- (a) $F = 10$ newton.
- (b) $F = 10\sqrt{2}$ newton.
- (c) $F = 5\sqrt{2}$ newton.
- (d) the system can not be in equilibrium.



2 Answer the following questions :

- (1) Three coplanar forces of magnitudes 1, 2, $\sqrt{3}$ newton act at M, their directions are $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ respectively where $m(\angle AMB) = 60^\circ$, $m(\angle BMC) = 30^\circ$, $m(\angle AMC) = 90^\circ$ Find the resultant. (2 marks)
- (2) $\overline{AB}$ is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and with point C on the wall above A and at a distance 60 cm. from A. Find the magnitude of the tension in the string and magnitude of the reaction of the hinge at A. (2 marks)

1 Choose the correct answer from the given ones :

(6 marks)

(1) The resultant of two forces of magnitudes 6, 8 newton is 10 N. , then the measure of the angle between their directions =°

- (a) 60 (b) 90 (c) 120 (d) 150

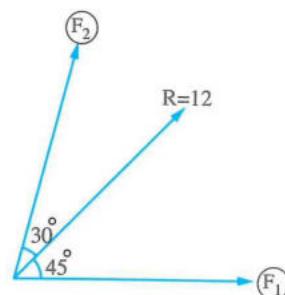
(2) Two forces intersecting at a point , their magnitudes 7 and F newton and their resultant bisects the angle between them , then $(F - 1) = \dots\dots\dots$ N.

- (a) 8 (b) 7 (c) 6 (d) 5

(3) In the opposite figure :

The force $\vec{R}$ is resolved into two components $\vec{F}_1$ and $\vec{F}_2$, then $F_1 = \dots\dots\dots$ newton.

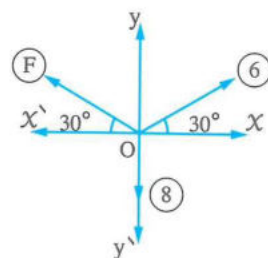
- (a) $12 \cos 75^\circ$ (b) $12 \cos 45^\circ$
(c) $6 \csc 45^\circ$ (d) $6 \csc 75^\circ$



(4) In the opposite figure :

If the resultant of the shown forces acts in direction of y-axis , then $F = \dots\dots\dots$ N.

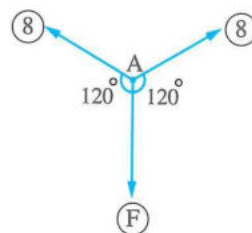
- (a) 2 (b) 6
(c) 8 (d) 14



(5) In the opposite figure :

Particle A is kept in equilibrium under action of the three forces , as shown in the figure , where $\vec{F}$ is in equilibrium with two forces each of magnitude 8 N. and it makes with each an angle of measure 120° , then $F = \dots\dots\dots$ N.

- (a) zero (b) 8 (c) 16 (d) $8 \sin 120^\circ$



(6) A body of weight (W) newton is placed on a smooth plane inclined with the horizontal at an angle of measure 30° and kept in equilibrium by a force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards , then the magnitude of the weight = newton.

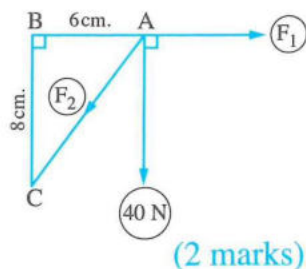
- (a) 36 (b) $72\sqrt{3}$ (c) 72 (d) $36\sqrt{3}$

2 Answer the following questions :

(1) In the opposite figure :

If the force of magnitude 40 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$ as shown in the figure.

Find the two component magnitudes F_1 , F_2



(2 marks)

- (2)** A weight of magnitude 200 gm.wt. is suspended by two strings of lengths 60 cm. and 80 cm. , from two points on the same horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string.

(2 marks)

Test

1

Total mark

10

1 Choose the correct answer from those given :

(6 marks)

- 1 Two forces of magnitudes 8 and 16 kg.wt. and the measure of their included angle is 120° . If these two forces act at a body, then the direction of motion of the body makes an angle of measure with the smaller force.
 (a) 30° (b) 90° (c) 60° (d) 45°
- 2 Two forces of equal magnitude and intersecting at a point. The measure of the angle between the two forces is 120° and the magnitude of each is 6 N., then the magnitude of their resultant = N.
 (a) 12 (b) $6\sqrt{3}$ (c) 6 (d) $12\sqrt{3}$
- 3 F N. and K N. are the magnitudes of two forces where $F > K$. If the smallest and the greatest value of their resultant are 5, 9 newton respectively, then $5F - 2K = \dots\dots\dots$ N.
 (a) 53 (b) 31 (c) 49 (d) 4
- 4 A body of weight 20 N. is placed on a smooth inclined plane makes an angle of measure 30° with the horizontal, then the component of the weight in direction perpendicular to the plane = N.
 (a) 10 (b) 20 (c) $10\sqrt{2}$ (d) $10\sqrt{3}$
- 5 Forces of magnitudes 8, $4\sqrt{3}$, $6\sqrt{3}$, 14 newton act at a point. The measure of the angle between the first and second force is 30° and between the second and third is 120° and between the third and fourth is 90° in one cyclic order, then the magnitude of their resultant =
 (a) 4 (b) 6 (c) 8 (d) 7
- 6 Two forces of magnitudes 3, F newton and measure of the angle between them is $\frac{2\pi}{3}$ if their resultant is perpendicular to the first force, then $F = \dots\dots\dots$ newton.
 (a) 1.5 (b) 3 (c) $\sqrt[3]{2}$ (d) 6

2 Answer the following questions :

- 1 A force of magnitude 18 newton acts in south direction. Find its two components in directions of 60° East of South and 30° West of South. (2 marks)
- 2 Three coplanar forces of magnitudes 1, 2, $\sqrt{3}$ newton act at M, their directions are $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ respectively where $m(\angle AMB) = 60^\circ$, $m(\angle BMC) = 30^\circ$, $m(\angle AMC) = 90^\circ$. Find the resultant. (2 marks)

Test

2

Total mark

10

1 Choose the correct answer from those given :

(6 marks)

- 1 The resultant of two forces 6 , 8 newton is 10 N. , then the measure of the angle between their directions =°

(a) 60 (b) 90 (c) 120 (d) 150

- 2 Two forces intersecting at a point , their magnitudes 7 and F newton and their resultant bisects the angle between them , then $(F - 1) = \dots\dots\dots$ N.

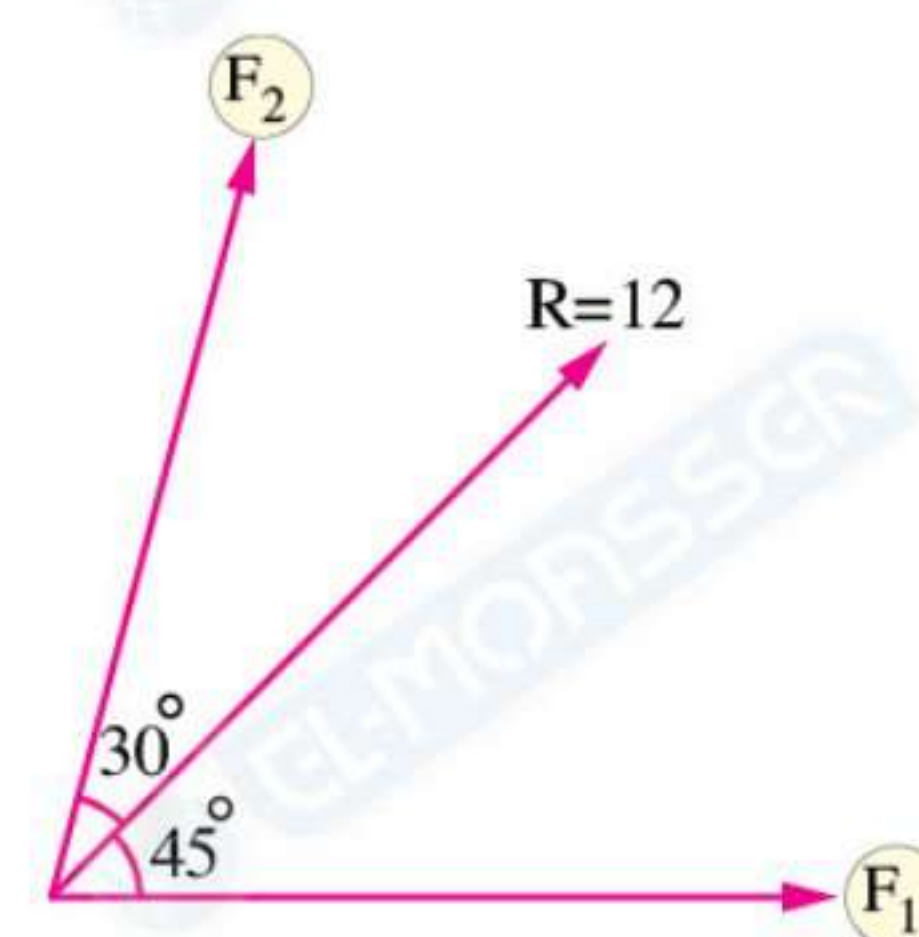
(a) 8 (b) 7 (c) 6 (d) 5

- 3 In the opposite figure :

The force $\vec{R}$ is resolved into two components $\vec{F}_1$ and $\vec{F}_2$

, then $F_1 = \dots\dots\dots$ newton.

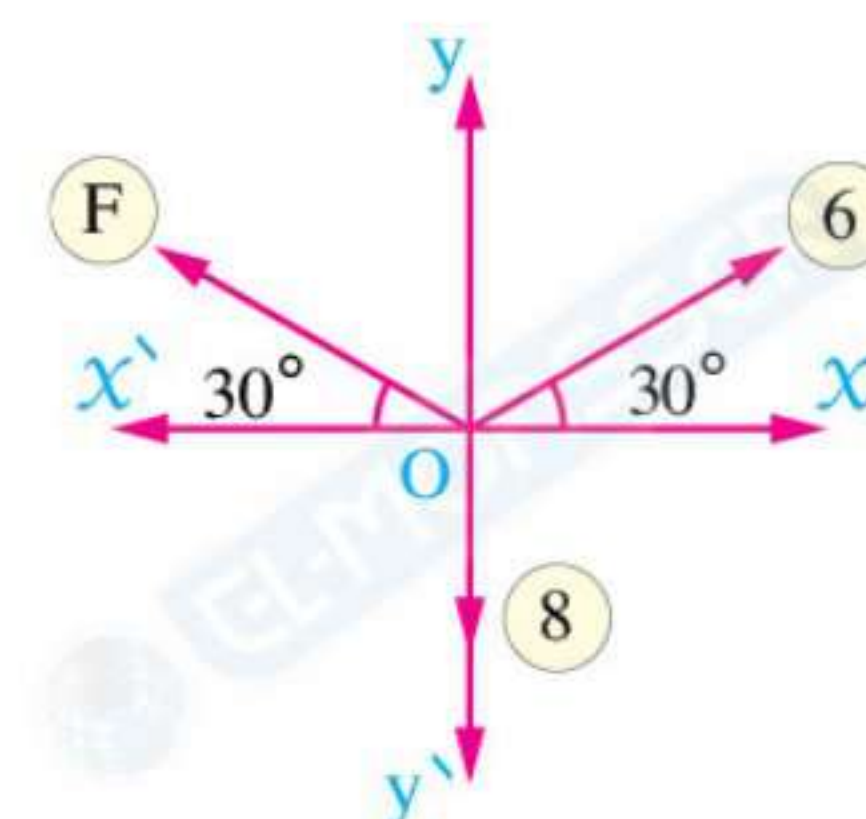
(a) $12 \cos 75^\circ$ (b) $12 \cos 45^\circ$
(c) $6 \csc 45^\circ$ (d) $6 \csc 75^\circ$



- 4 In the opposite figure :

If the resultant of the shown forces acts in direction of y-axis , then $F = \dots\dots\dots$ N.

(a) 2 (b) 6
(c) 8 (d) 14



- 5 The magnitudes of two forces are 5 and 10 newton and their resultant is perpendicular on the smaller force. If the measure of angle between the two forces is α and their resultant is R , then

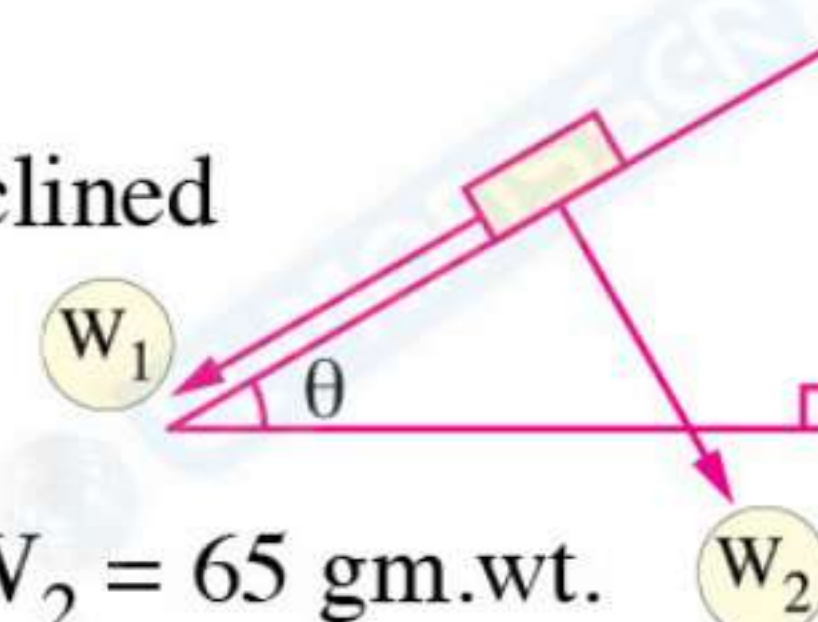
(a) $\alpha = 60^\circ$, $R = 10\sqrt{3}$ N. (b) $\alpha = 120^\circ$, $R = 10\sqrt{3}$ N.
(c) $\alpha = 60^\circ$, $R = 5\sqrt{3}$ N. (d) $\alpha = 120^\circ$, $R = 5\sqrt{3}$ N.

- 6 In the opposite figure :

A body of wiehgt 260 gm.wt. and $\tan \theta = \frac{5}{12}$,

W_1 , W_2 are magnitudes of the two components in direction of the inclined plane downward and perpendicular to the plane , then

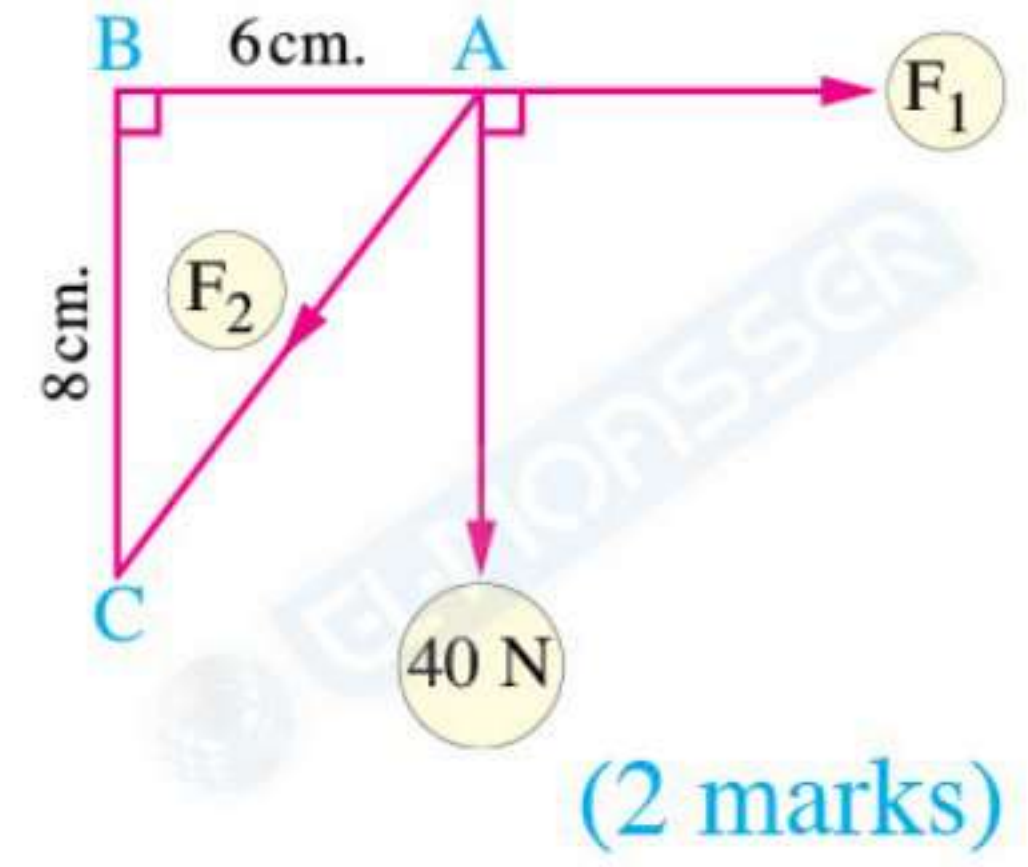
(a) $W_1 = 120$ gm.wt. , $W_2 = 50$ gm.wt. (b) $W_1 = 260$ gm.wt. , $W_2 = 65$ gm.wt. (c) $W_1 - W_2 = 70$ gm.wt. (d) $W_1 + W_2 = 340$ gm.wt.



2 Answer the following questions :**1 In the opposite figure :**

If the force of magnitude 40 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$ as shown in the figure.

Find the two component magnitudes F_1 , F_2



- 2** The magnitudes of three forces are 10 , 20 , 30 newton acting at one point. The first acts due east , the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° south of the west. Find the magnitude and the direction of their resultant.
- (2 marks)

Answers of Test

1

- 1 1 (b) 2 (c) 3 (b) 4 (d) 5 (a) 6 (d)

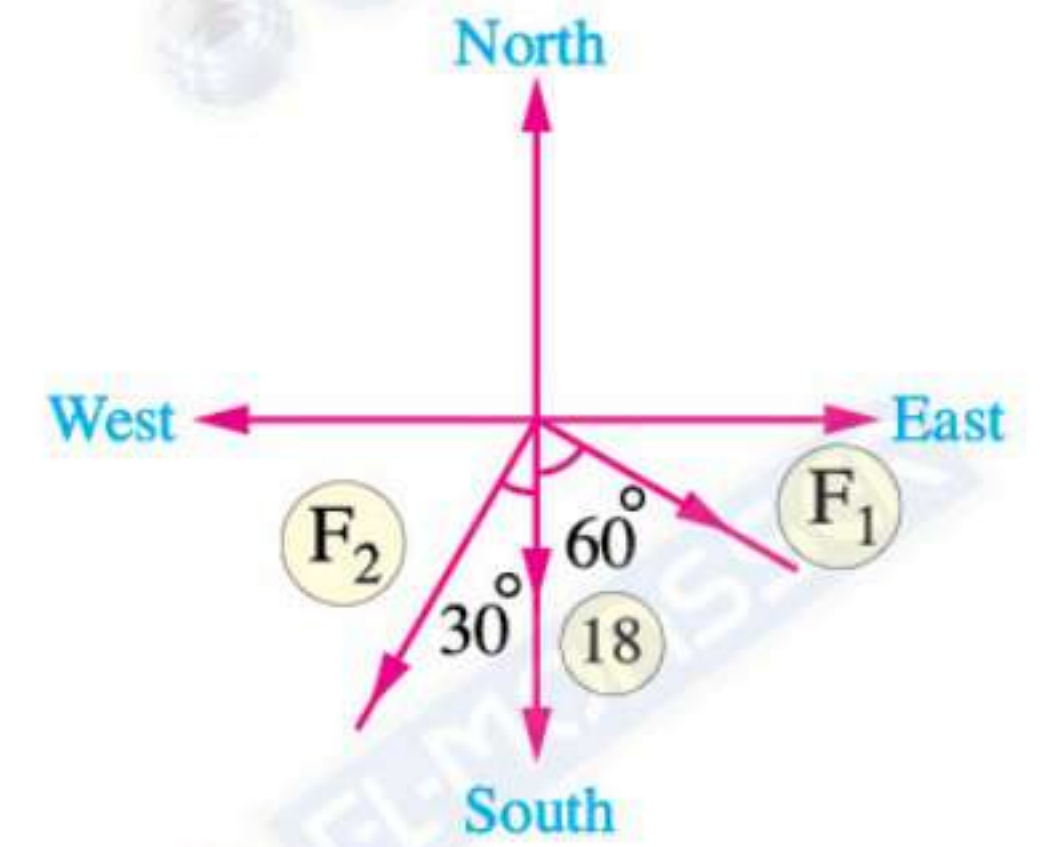
2 1 ∴ The two components are perpendicular

$$\therefore F_1 = 18 \cos 60^\circ$$

$$= 9 \text{ newton}$$

$$, F_2 = 18 \sin 60^\circ$$

$$= 9\sqrt{3} \text{ newton}$$



2 Consider $\vec{OX}$ is the direction of the first force.

$$X = 1 \times \cos 0^\circ + 2 \cos 60^\circ + \sqrt{3} \cos 90^\circ$$

$$= 1 \times 1 + 2 \times \frac{1}{2} + \sqrt{3} \times 0 = 2$$

$$Y = 1 \times \sin 0^\circ + 2 \times \sin 60^\circ + \sqrt{3} \sin 90^\circ$$

$$= 1 \times 0 + 2 \times \frac{\sqrt{3}}{2} + \sqrt{3} \times 1 = 2\sqrt{3}$$

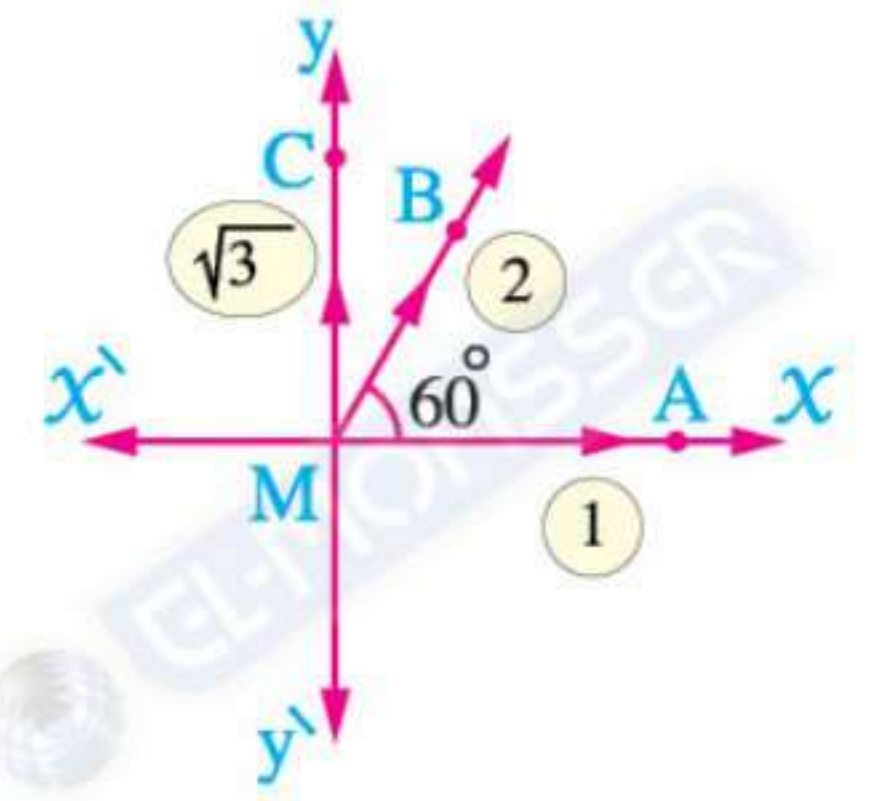
$$\therefore \vec{R} = 2\vec{i} + 2\sqrt{3}\vec{j}, R = \sqrt{(2)^2 + (2\sqrt{3})^2} = 4 \text{ newton}$$

$$, \tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$, \therefore X > 0, Y > 0$$

$$\therefore \theta = 60^\circ$$

∴ The magnitude of $\vec{R} = 4$ newton and its direction is $\vec{MB}$



Answers of Test

2

- 1 1 (b) 2 (c) 3 (d) 4 (b) 5 (d) 6 (d)

2 1 From the figure

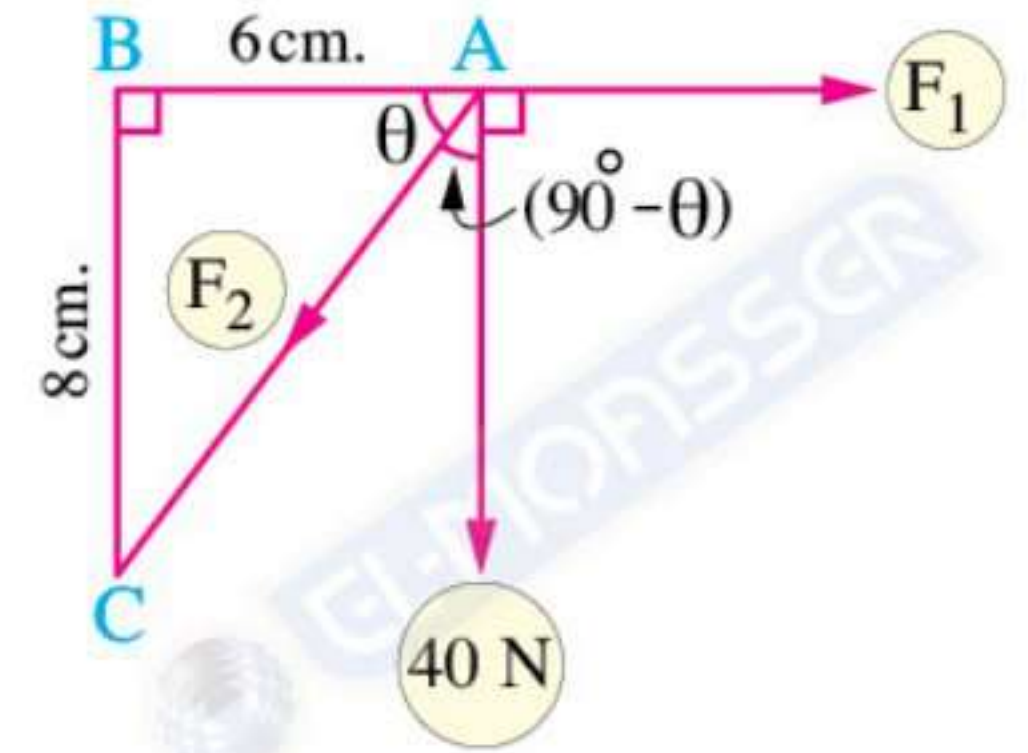
$$\sin \theta = 0.8, \cos \theta = 0.6$$

$$\therefore \frac{F_1}{\sin (90^\circ - \theta)} = \frac{F_2}{\sin 90^\circ} = \frac{40}{\sin (180^\circ - \theta)}$$

$$\therefore \frac{F_1}{\cos \theta} = \frac{F_2}{1} = \frac{40}{\sin \theta}$$

$$\therefore \frac{F_1}{0.6} = \frac{F_2}{1} = \frac{40}{0.8}$$

$$\therefore F_1 = 30 \text{ N.}, F_2 = 50 \text{ N.}$$



$$2 \quad X = 10 \cos 0^\circ + 20 \cos 120^\circ + 30 \cos 240^\circ = -15$$

$$Y = 10 \sin 0^\circ + 20 \sin 120^\circ + 30 \sin 240^\circ = -5\sqrt{3}$$

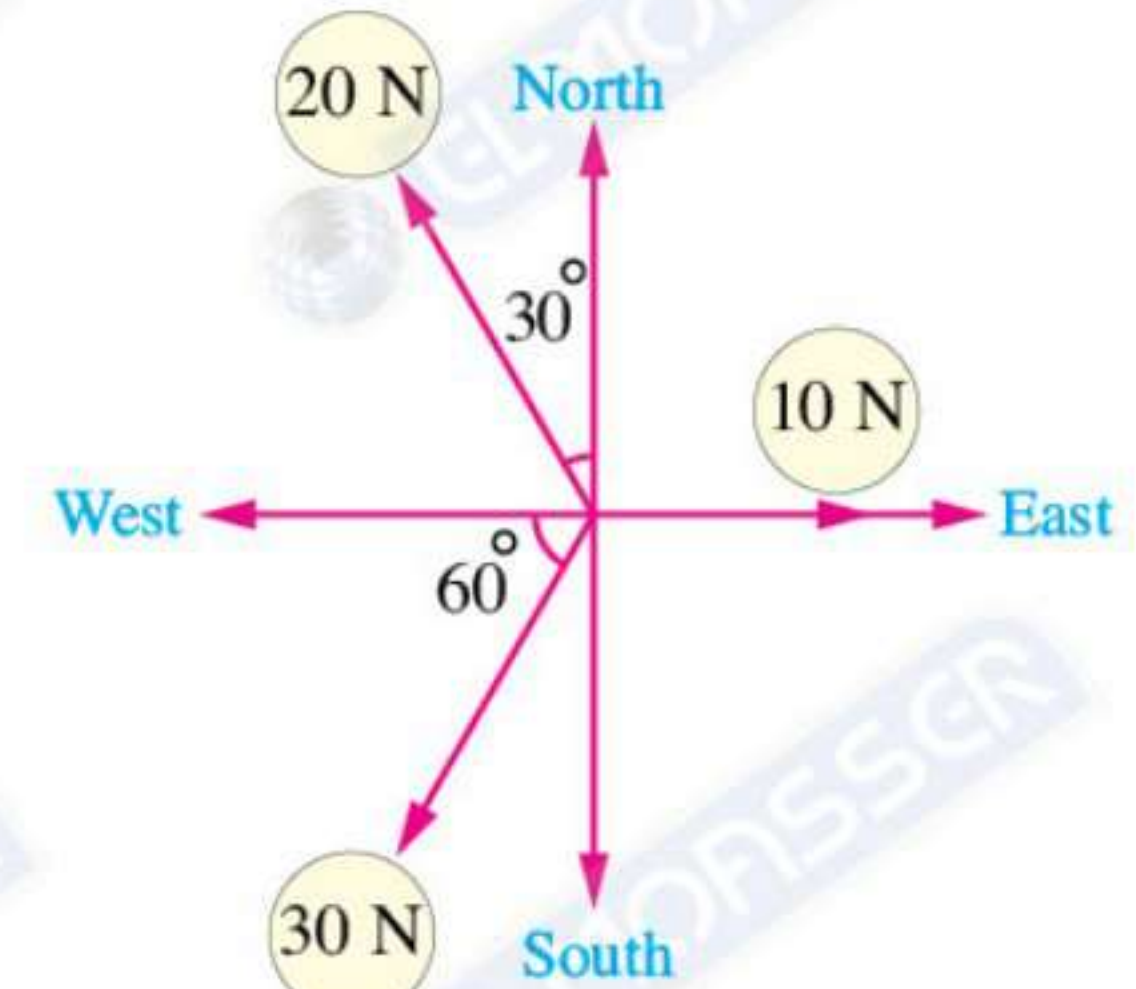
$$\therefore \vec{R} = -15\vec{i} - 5\sqrt{3}\vec{j}$$

$$\therefore R = \sqrt{225 + 75} = 10\sqrt{3} \text{ N.}$$

$$\tan \theta = \frac{y}{x} = \frac{-5\sqrt{3}}{-15} = \frac{1}{\sqrt{3}}$$

$$\therefore x < 0, y < 0$$

$$\therefore \theta = 180^\circ + 30^\circ = 210^\circ$$



كيفية طباعة صفحات معينة من ملف معين مثلا ازاي نطبع الصفحات من صفحة 4 الى صفحة 9



حمل الآن

مجاناً وحصرياً

المراجعة رقم (2)

اختبار شهر اكتوبر



Lesson (1): Forces

The force

"The force is defined as the effect of a natural body upon another one" by pushing, attraction, pressure or repulsion".

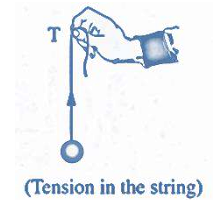
The natural body is a body consisting of material (mass) and volume not equal to zero,

Kinds of forces

There are different kinds of forces? as:

1) Tension force (T):

As the force in the string (or the rope) when carrying a body at it.

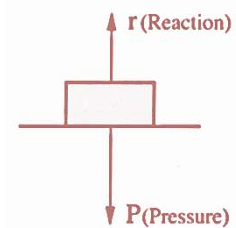


2) Pressure force (P):

As the force that appears when a body stabilizes on a surface.

3) Reaction force (r):

As the reaction of a smooth surface on a body stabilized on it.



4) Attraction forces and repulsion forces:

As the forces which formed between magnetic poles, electric charges and astronomical objects.

5) Gravitational forces (weights):

If we let a body in the air, then it will drop down towards the Earth because the attraction force of the Earth attracts anybody towards it.

This force is called Gravitational force or weights.

Notice that: the weight (W) = the body mass x acceleration of gravity = $m \times g$

The force can be expressed as follow:

- (1) $\vec{F} = (X, y)$ $\Rightarrow$ the cartesian form .
- (2) $\vec{F} = X \vec{i} + y \vec{j}$ $\Rightarrow$ in term s of the fundamental unit vectors.
- (3) $\vec{F} = (\parallel \vec{F} \parallel, \theta)$ $\Rightarrow$ The polar form .

The measurement units:

- 1) 1 kg.wt. = 1000 gm.wt. = 103 gm.wt.
- 2) 1 newton = 100 000 dyne - 105 dyne
- 3) 1 kg.wt. = 9.8 newton , 1 gm.wt. = 980 dyne

Main Points

⇒ **Statics:** The branch of mechanics that deals with bodies at rest or forces in equilibrium.

⇒ **Force:** The effect of a natural body upon another one.

⇒ **Properties of a force:** The effect of a force is determined by:

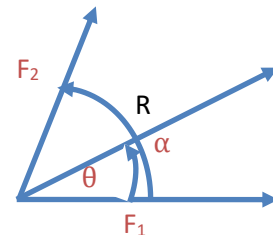
- 1- Magnitude. 2- Direction. 3- Point of action. (Line of action)

If $\vec{F}_1$, $\vec{F}_2$ are two forces , the angle between their lines of actions (α) , their resultant $\vec{R}$ which inclined by an angle of measure (θ) with the direction of $\vec{F}_1$ then:

&

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha} \quad (\text{Magnitude})$$

$$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} \quad (\text{Direction})$$

Special cases:

i) If $F_1 \perp F_2 \Rightarrow$

$$R = \sqrt{F_1^2 + F_2^2}$$

, and

$$\tan \theta = \frac{F_2}{F_1}$$

ii) If $F_1 = F_2 = F \Rightarrow \theta = \frac{\alpha}{2}$ and R bisects the angle between F_1 and F_2 ,

$$R = 2F \cos \frac{\alpha}{2}$$

iii) if $\alpha = 0$ then $R = F_1 + F_2$ (maximum resultant)

iv) if $\alpha = 180$ then $R = |F_1 - F_2|$ (minimum resultant)

v) If $\theta = 90^\circ \Rightarrow F_1 + F_2 \cos \alpha = 0$ and $R = \sqrt{F_2^2 - F_1^2}$

Note: If we want the measure of the angle between R and F_2 , Find $(\alpha - \theta)$

Or from the law:

$$\tan \theta = \frac{F_1 \sin \alpha}{F_2 + F_1 \cos \alpha}$$

1) Two forces of magnitudes 5 newton and 3 newton act at a point and include an angle of measure 60° ? find their resultant in magnitude and direction analytically.

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha}$$

$$R = \sqrt{(5)^2 + (3)^2 + 2 \times 5 \times 3 \times \cos 60} = 7 \text{ newton.}$$

$$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\tan \theta = \frac{3 \sin 60}{5 + 3 \cos 60} = \frac{3\sqrt{3}}{13}$$

$$\theta = 21^\circ 47'$$

The magnitude of R is 7 newton and include an angle of measure $21^\circ 47'$ with the first force.

2) Two perpendicular forces act at a point such that $F_1 = 6$ newton and $F_2 = 2.5$ newton. Find their resultant in magnitude and find its direction.

$$R = \sqrt{F_1^2 + F_2^2}$$

$$R = \sqrt{6^2 + 2.5^2} = 6.5 \text{ newton.}$$

$$\tan \theta = \frac{F_2}{F_1}$$

$$\tan \theta = \frac{2.5}{6} = \frac{5}{12} \quad \theta = 22^\circ 37'$$

The magnitude of R is 6.5 newton and include an angle of measure $22^\circ 37'$ with the first force.

3) Two forces of magnitudes 50 newton and 100 newton act at a point. Their resultant is perpendicular to the first force. Find the measure of the angle included between the two forces and the magnitude of the resultant.

$F_1 = 50$ newton , $F_2 = 100$ newton.

The resultant is perpendicular to the first force.

$$F_1 + F_2 \cos \alpha = 0$$

$$50 + 100 \cos \alpha = 0$$

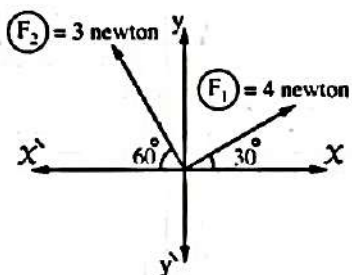
$$\cos \alpha = \frac{-1}{2}$$

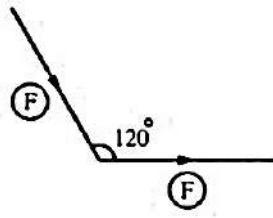
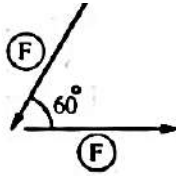
$$\alpha = 120^\circ$$

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha}$$

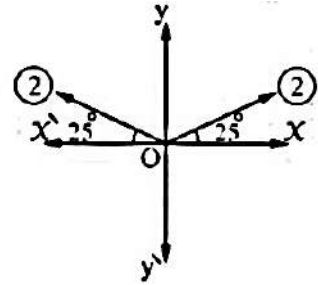
$$R = \sqrt{(50)^2 + (100)^2 + 2 \times 50 \times 100 \times \cos 120} = 50\sqrt{3} \text{ newton.}$$

[1] Choose the correct answer:

1)	The force is defined by	
	(a) its magnitude. (b) its direction. (c) the point of action. (d) all the previous.	
2)	Two forces act at a point. The magnitude of the two forces are 5 , 3 newton and the angle between them 60° , then the magnitude of their resultant = newton. (a) 2 (b) 5 (c) 7 (d) 8	
3)	Two perpendicular forces act at a point. The magnitude of the two forces 12 , 5 newton , then the magnitude of their resultant = newton. (a) 17 (b) 7 (c) 13 (d) 14	
4)	Resultant of two forces 6 newton and 8 newton could be newton. (a) 20 (b) 15 (c) 12 (d) 1	
5)	The magnitude of two forces are 4 , 5 N. They act at a point and cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant R = newtons. (a) 15 (b) 5 (c) 20 (d) 25	
6)	Two forces act at a point. The magnitude of the two forces are 6 , 3 newton and their resultant is perpendicular to one of them , then the magnitude of their resultant = newton. (a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$	
7)	In the opposite figure : The magnitude of the resultant of the two forces in the figure = newton. (a) 7 (b) 5 (c) 1 (d) $\sqrt{7}$	

8)	In the opposite figure : Magnitude of the resultant of the two forces = newton. (a) $2F$ (b) F (c) $\sqrt{3}F$ (d) zero	
9)	The magnitude of the resultant of the two forces shown in the opposite figure is (a) $\frac{1}{2}F$ (b) F (c) $\sqrt{3}F$ (d) $\sqrt{5}F$	
10)	Two forces act at a point. The magnitude of the two forces are F , 2 newton and the measure of the angle between them is 60° , if their resultant equal $2\sqrt{3}$ newton , then $F = \dots\dots\dots$ newton. (a) 2 (b) 4 (c) 8 (d) 12	
11)	The magnitude of two forces F , 2 newton and the measure of their included angle $= \frac{2\pi}{3}$ and the magnitude of their resultant is F newton , then $F = \dots\dots\dots$ newton. (a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$	
12)	 Two forces of equal magnitudes , enclosing between them an angle of measure $\frac{\pi}{2}$ If the magnitude of their resultant is 8 N. , then the value of each force measured in newton is (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8	
13)	The magnitude of two forces F , F kg.wt. , the magnitude of their resultant 24 kg.wt. and inclined to the first force by an angle of measure 30° , then $F = \dots\dots\dots$ kg.wt. (a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12	

14)	Two forces of magnitudes 8 and F gm.wt. The measure of the angle between them is $\alpha \in]0, \pi[$, their resultant bisects the included angle between them, then F = gm.wt. (a) 4 (b) 16 (c) $2\sqrt{2}$ (d) 8
15)	 Two forces of magnitudes 3, F newton and the measure of the angle between them is 120° . If their resultant is perpendicular to the first force, so the value of F in newton is (a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6
16)	 Two equal forces, the magnitude of each of them is 6 N., the magnitude of their resultant is 6 N., then the angle between them equals (a) 30° (b) 60° (c) 120° (d) 150°
17)	Two forces of magnitudes 6 N. and 8 N., if the magnitude of their resultant is 2 N., then the measure of the angle between the two forces is (a) 30° (b) 90° (c) 180° (d) 270°
18)	Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton, then the measure of the angle between them = (a) 60° (b) 0° (c) 180° (d) 90°
19)	Two forces act at a point. The magnitude of the two forces are 5 F, 3 F. If the maximum value of their resultant is 40 newton, then the minimum value of their resultant newton. (a) 10 (b) 20 (c) 5 (d) zero
20)	Two forces act at a point. The magnitudes of the two forces are 5, 3 newton, then the magnitude of their resultant measure by newton $\in$ (a) $[2, 8]$ (b) $]2, 8[$ (c) $[3, 5]$ (d) $]3, 5[$

21)	The magnitude of two forces are 12 , 17 newton then the difference between the greatest and the smallest value of their resultant = newton. (a) 29 (b) 5 (c) 14 (d) 24
22)	The direction of the resultant of the forces which represented in the opposite figure is (a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$ 
23)	Two forces of magnitude F_1 , F_2 kg.wt. , where $F_1 > F_2$ and the magnitude of smallest and greatest resultant of them are 3 and 12 gm.wt. respectively , then $F_1^2 - F_2^2 = \dots\dots\dots$ (a) 12 (b) 3 (c) 9 (d) 36
24)	The magnitudes of two perpendicular forces are 6 , 8 newton then the measure of the angle between the resultant and the first force is (a) $\sin^{-1} \frac{4}{3}$ (b) $\cos^{-1} \frac{4}{3}$ (c) $\tan^{-1} \frac{4}{3}$ (d) $\tan^{-1} \frac{3}{4}$

21 Answer the following questions:

[1] Two forces of magnitudes 5, 10 Newton act on a particle and the measure of the angle between them is 120° . Find the magnitude of their resultant and the measure of the angle made by the resultant with the first force.

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[2] Two forces of magnitudes $8\sqrt{3}$, 8 Newton act on a particle and the measure of the angle between them is 150° . Find the magnitude of their resultant and the measure of the angle which it makes with the first force

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[3] Two forces of magnitudes 30, 16 Newton act on a particle. If their resultant equals 26 Newton, find the measure of the angle between the two forces.

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[4] Two forces of magnitudes 12 , 15 Newton act on a particle and the (cosine) of the angle between them equals $\frac{-4}{5}$,Find the magnitude of their resultant and the measure of the angle of inclination of the resultant to the first force.

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[5] Three forces of magnitude 5, 10 and $4\sqrt{4}$ Newton act on a particle, if the measure of the angle between the first and the second forces equals 60° , find the magnitude of the maximum and the minimum resultant for the three forces.

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[6] Two forces of magnitudes 4 , F Newton act on a particle and the angle between them is 120° . If their resultant is perpendicular to the first force, find the magnitude of F

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[7] Two forces act at a point, If the magnitude of their maximum resultant equals 32 kg.wt and the magnitude of their minimum resultant equals 12 kg.wt , Find the magnitude of each of the two forces ,then find their resultant if they enclosed between them an angle of measure 60° .

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[8] Two forces of magnitude $F, F\sqrt{2}$ N act on a particle; if their resultant is perpendicular to the first force, Find the angle between the two forces then prove that the magnitude of their resultant equals F .

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[9] Two forces of magnitudes $2, F$ Newton, the angle between them is of measure 120° find F in each of the following cases:

- (i) magnitude of the resultant = F .
- (ii) the direction of the resultant is perpendicular to the second force
- (iii) the resultant bisect the angle between the angle of two forces
- (iv) the resultant inclines by 45° to the second force

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[10] Two forces of same magnitude F kg.wt enclose between them an angle of measure 120° . If the two forces are doubled and the measure of the angle between them became 60° ,then the magnitude of their resultant increases by 11 k.g.wt than the first case . Find the magnitude of F

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[11] Two forces of magnitudes 12 , F kg.wt act on a point. The first force acts in direction of east and the second force acts in direction 60° south of the west. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° south of the east.

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[12] F1 , F2 are two forces act on a particle and enclose between them an angle of measure 120° and the magnitude of their resultant is $\sqrt{19}$ N, if the angle between them becomes 60° ,then the magnitude of their resultant becomes 7 newton. Find the value of each of F1 , F2

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[13] Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if t he direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. find the magnitude of each force.

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Lesson (2): Forces resolution into two components

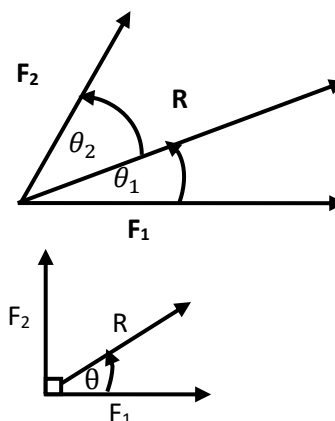
Main Points

Resolution of a force into two components:

⇒ (i) If $\vec{F}_1$, $\vec{F}_2$ are two components of a force $\vec{R}$, they make with the direction of $\vec{R}$ two angles of measures θ_1 , θ_2 respectively, then:

$$\frac{F_1}{\sin \theta_2} = \frac{F_2}{\sin \theta_1} = \frac{R}{\sin(\theta_1 + \theta_2)}$$

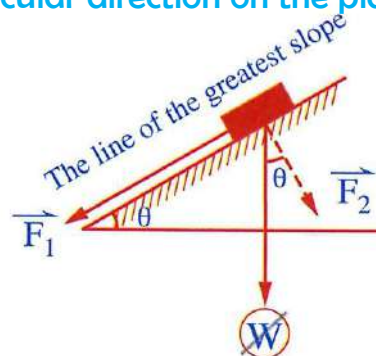
ii) If $F_1 \perp F_2 \Rightarrow F_1 = R \cos \theta$, $F_2 = R \sin \theta$



Note: 1) If $\vec{F} = (F, \theta)$, then $\vec{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}$

2) If a body of weight (w) is placed on a smooth inclined plane with the horizontal by an angle (θ), then we can resolve the weight (w) which acts vertically downwards into two components.

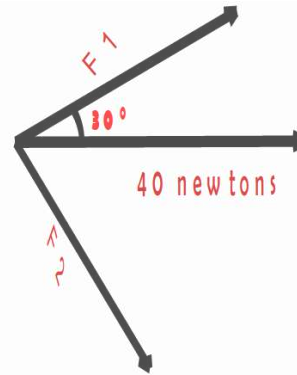
- ★ F_1 (The magnitude of the component in the direction of the line of the greatest slope)
= $w \sin \theta$
- ★ F_2 (The magnitude of the component in the perpendicular direction on the plane)
= $w \cos \theta$



1) Resolve a horizontal force 40 newtons into two perp. directions one of them inclines by 30° with the horizontal upwards

$$F_1 = 40 \cos 30 = 20\sqrt{3} \text{ N}$$

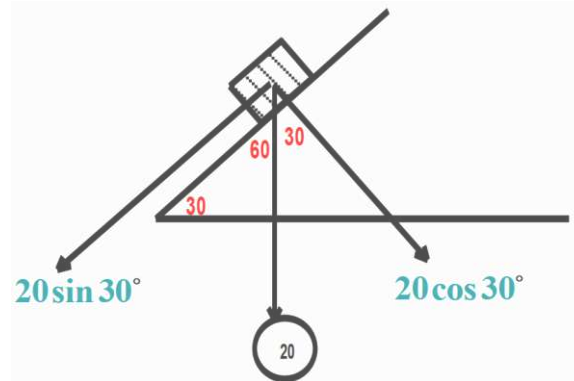
$$F_2 = 40 \sin 30 = 20 \text{ N}$$



2) A body of weight 20 newtons is placed on an inclined plane of inclination 30° to the horizontal. Find the components of the weight in the direction of the plane and the normal to it

$$F_1 = R \cos \theta = 20 \cos 30^\circ = 10\sqrt{3} \text{ newtons}$$

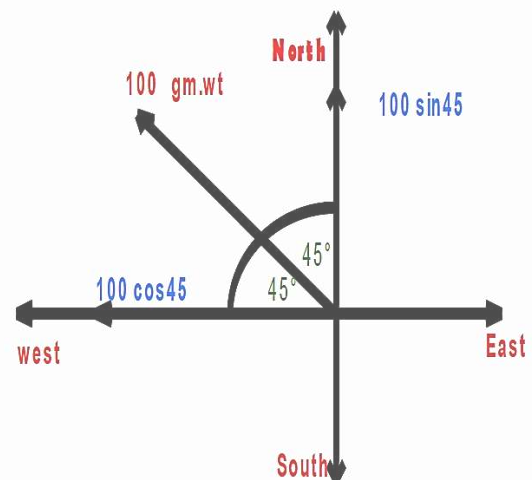
$$F_2 = R \sin \theta = 20 \sin 30^\circ = 10 \text{ newtons}$$



3) A Force of magnitude 100 gm.wt acts in the directions north west. find its components due to north and west.

$$F_1 = R \cos \theta = 100 \cos 45^\circ = 50\sqrt{2} \text{ newtons}$$

$$F_2 = R \sin \theta = 100 \sin 45^\circ = 50\sqrt{2} \text{ newtons}$$



Complete each of the following:

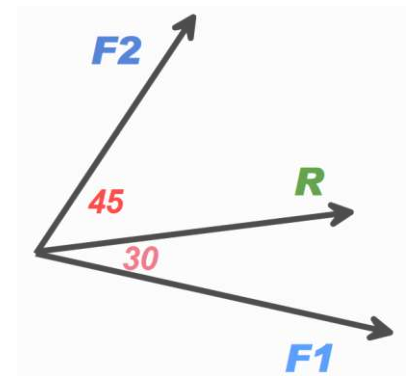
[1] A force of magnitude 6 Newton acts in direction of North. It is resolved into two perpendicular components, so its component in direction of the East equals Newton.

[2] A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components, so its component in the direction of Northern East =..... Newton.

[3] In the opposite figure:

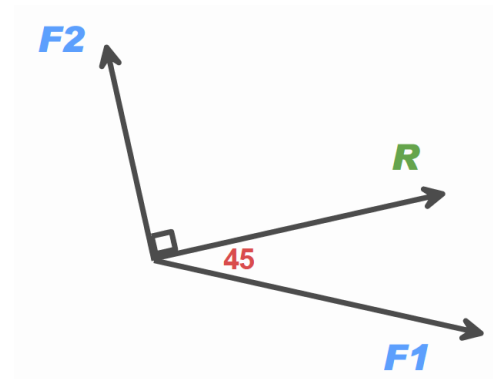
If the force $\vec{R}$ is resolved into two components $\vec{F}_1$, $\vec{F}_2$ which make with the force $\vec{R}$ two angles of measures 30° , 45° from different directions of its line of action, $||\vec{R}|| = 12$ newton,

So: $F_1 =$ Newton, $F_2 =$ Newton

[4] In the opposite figure:

If the force $\vec{R}$ is resolved into two components $\vec{F}_1$, $\vec{F}_2$ which make with the force $\vec{R}$ two angles of measure 45° , 90° from different directions of its line of action and $||\vec{R}|| = 18$ Newton,

So: $F_1 =$ Newton, $F_2 =$ Newton



[5] In the opposite figure:

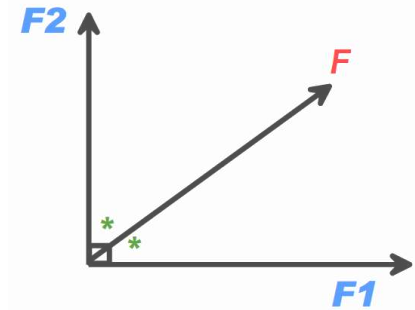
If the force $\vec{F}$ is resolved into two perpendicular components

$\vec{F}_1$, $\vec{F}_2$ and the force vector $\vec{F}$ bisects the angle between

The directions of $\vec{F}_1$, $\vec{F}_2$ and $\|\vec{F}\| = 6\sqrt{2}$ kg. wt

so: $\|\vec{F}_1\| = \dots\dots\dots$ kg wt ,

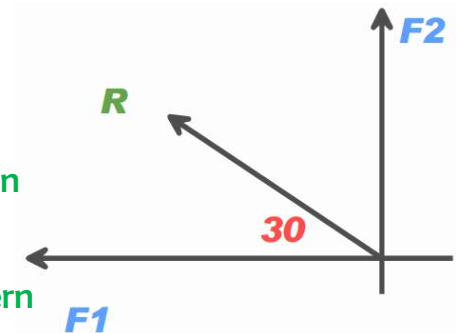
$\|\vec{F}_2\| = \dots\dots\dots$ kg wt.



[6] In the opposite figure:

Force of magnitude $12\sqrt{2}$ newton acts in direction 30° North of the west.

- Magnitude of the component of the force in the western direction = $\dots\dots\dots$ Newton.
- Magnitude of the component of the force in the northern direction = $\dots\dots\dots$ Newton.



[7] A force of magnitude 600 gm.wt acts on a particle. Find its two components in two directions making with the force two angles of measures 30° , 45° .

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[8] A force of magnitude 120 newton acts in direction of the North east. Find its two components in the direction of East and in the direction of North.

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[9] Resolve a horizontal force of magnitude 160 gm.wt in two perpendicular directions one of them inclined to the horizontal with an angle of measure 30° upwards.

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[10] A force of magnitude 18 newton acts in the direction of South. Find its two components in the 2 directions, 60° East of the South and the other direction towards 30° West of the South.

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[11] A rigid body of weight 42 newton is placed on a plane inclined to the horizontal with an angle of measure 60° . Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it.

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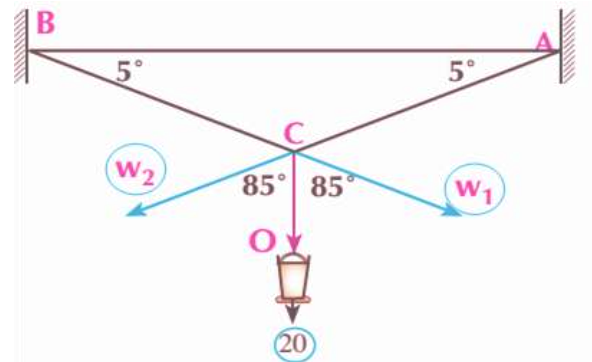
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[12] A lamp of weight 20 newton suspended by two metal rods $\overline{AC}$, $\overline{BC}$ inclined to the horizontal by two equal angles, the measure of each is 5° :
(1) Resolve the weight of the lamp into two components in the directions $\overline{AC}$, $\overline{BC}$ approximating the result to the nearest newton.



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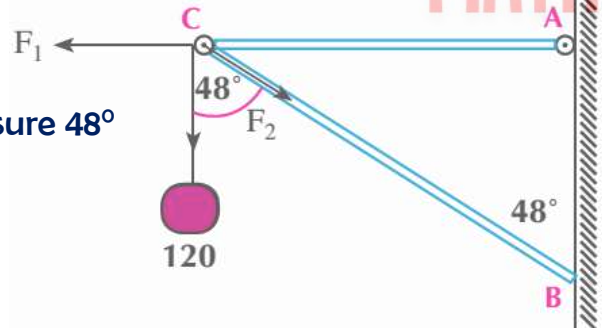
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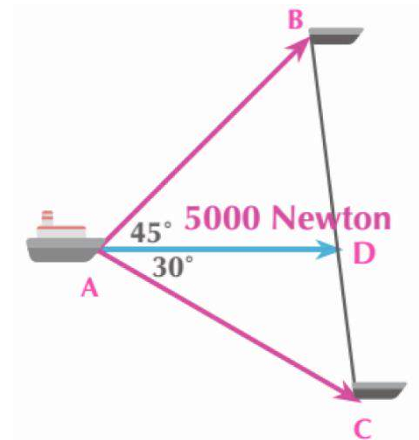
[13] In the opposite figure :

Resolve the vertical force of magnitude 120 gm.wt. into two components, one of them in the horizontal direction and the other inclined by an angle of measure 48° with the line of action of the force.



[14] In the opposite figure :

A cruiser is pulled by two ships B and C using two strands hanged to a point A on the cruiser, the measure of the angle between the two strands equals 75° , if the measure of the angle between one of the strands and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 newtons and acts on $\overrightarrow{AD}$ Find the tension in the two strands.

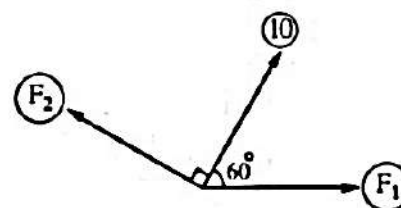


Choose the correct answer:

1)

In the opposite figure :

If the force of magnitude 10 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$ inclined to the force by two angles of measures 60° and 90° respectively , then $F_2 = \dots\dots\dots$ N.



(a) $5\sqrt{3}$

(b) 10

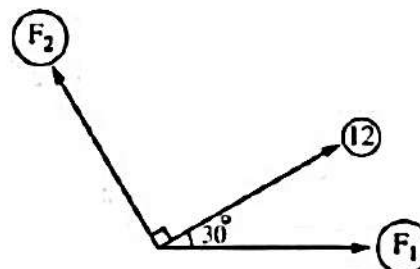
(c) $10\sqrt{3}$

(d) 20

2)

In the opposite figure :

If the force of magnitude 12 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$ inclined to the force by two angles of measures 30° and 90° respectively, then $F_2 = \dots\dots\dots$ N.



(a) 10

(b) $10\sqrt{3}$

(c) $6\sqrt{3}$

(d) $4\sqrt{3}$

3)

In the opposite figure :

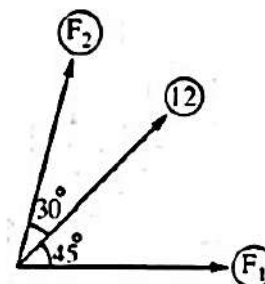
If the force of magnitude 12 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$, then $F_1 = \dots\dots\dots$ newton.

(a) $12 \cos 75^\circ$

(b) $12 \cos 45^\circ$

(c) $6 \csc 45^\circ$

(d) $6 \csc 75^\circ$



4)

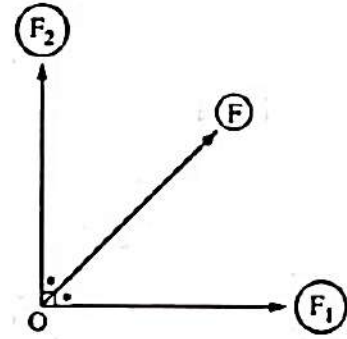
In the opposite figure :

If the force $\vec{F}$ is resolved into the two perpendicular components $\vec{F}_1$ and $\vec{F}_2$, the vector of the force $\vec{F}$ bisects the angle between the directions of $\vec{F}_1$ and $\vec{F}_2$ and $\|\vec{F}_1\| = 6\sqrt{2}$ newton, then $\|\vec{F}\| = \dots\dots\dots$ newton.

(a) 6

(b) $6\sqrt{2}$

(c) 12

(d) $12\sqrt{2}$ 

5)

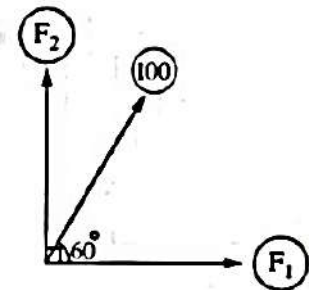
In the opposite figure :

If the force of magnitude 100 newton is resolved into two forces $\vec{F}_1$ and $\vec{F}_2$ and the force is measured by newton, then $(F_1, F_2) = \dots\dots\dots$

(a) $(50, 50\sqrt{3})$ (b) $(50\sqrt{3}, 10)$

(c) (50, 50)

(d) (10, 10)



6)

In the opposite figure :

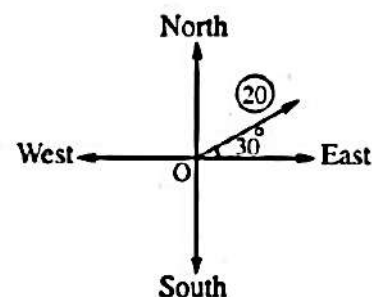
A force of magnitude 20 newton. acts in the direction 30° North of the East is resolved into two perpendicular components, then the magnitude of the component in North direction = $\dots\dots\dots$ newton.

(a) $10\sqrt{3}$

(b) 20

(c) 10

(d) 5



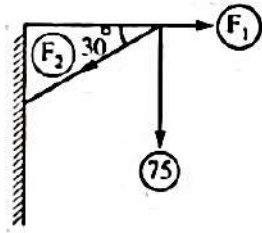
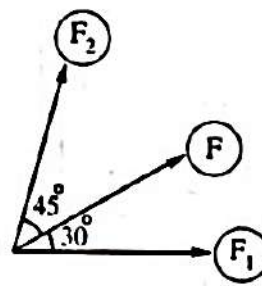
7)

A force of magnitude $10\sqrt{2}$ gm.wt. acts in the Eastern South direction, is resolved into two perpendicular components, then the magnitude of the component in the South direction = $\dots\dots\dots$ gm.wt.

(a) 5

(b) 10

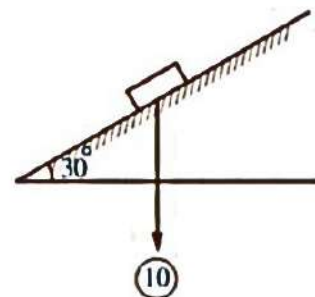
(c) $10\sqrt{2}$ (d) $5\sqrt{2}$

8)	📖 A force of magnitude 6 newton acts in direction of North. It is resolved into two perpendicular components , so its component in direction of the East of magnitude newton. (a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6	
9)	📖 A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components , so its component in the direction of Northern East of magnitude newton. (a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6	
10)	A force of magnitude 40 newton acts vertically upwards is resolved into two components one of them is horizontal of magnitude 20 newton , then the magnitude of the other = newton. (a) 20 (b) $20\sqrt{3}$ (c) $20\sqrt{5}$ (d) $10\sqrt{3}$	
11)	In the opposite figure : A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal of magnitude F_1 and the other is of magnitude F_2 , then $F_2 =$ newton. (a) 75 (b) $75\sqrt{3}$ (c) 150 (d) $150\sqrt{3}$	
12)	In the opposite figure : The force $\vec{F}$ is the resultant of the two forces $\vec{F}_1$, $\vec{F}_2$, then $\frac{F_1 + F_2}{F} =$ (a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$ (c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$	

13)

In the opposite figure :

If a body of weight 10 newtons is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , then the component of the weight in direction of line of the greatest slope downward = N.



(a) $5\sqrt{2}$

(b) $5\sqrt{3}$

(c) 5

(d) $10\sqrt{3}$

14)

If a body of weight (W) is placed on a smooth plane inclined to horizontal by angle (θ) , so the component of its weight in direction of the plane equals

(a) W

(b) $W \sin \theta$

(c) $W \cos \theta$

(d) $W \tan \theta$

15)

If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the horizontal , then its weight component in the perpendicular direction of the plane is

(a) $W \sin \theta$

(b) $W \cos \theta$

(c) $W \tan \theta$

(d) $W \csc \theta$

16)

A body of weight (W) newton is placed on an inclined plane makes an angle of measure (θ) with the horizontal , then the components of its weight in direction line of greatest slope and its perpendicular are 7 , 24 newton respectively , then the magnitude of the weight (W) = newton.

(a) 7

(b) 24

(c) 25

(d) 31

Lesson 3: The resultant of coplanar forces meeting at a point

Main Points

- **Polygon of forces:** If a set of coplanar forces meeting at a point are represented by sides of a polygon taken in the same cyclic order, then the magnitude of the resultant of these forces equals the length of side that closed this polygon in the opposite cyclic order.
- In a perpendicular coordinate plane if a set of coplanar forces that meeting at a point at this point, and the algebraic sum of the components of these forces in the two perpendicular direction are X, Y ,

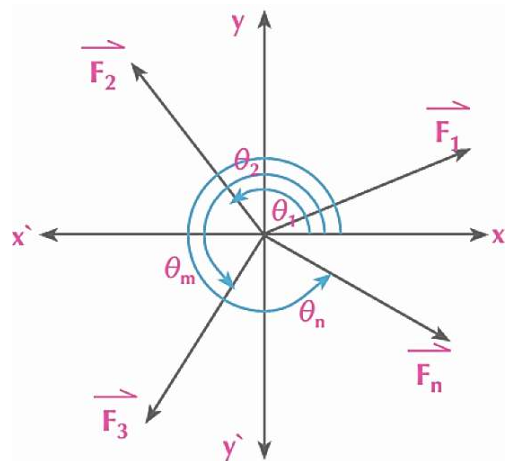
Where $X = \sum F_i \cos \theta_i$ and $Y = \sum F_i \sin \theta_i$

then:

$$R = \sqrt{X^2 + Y^2}$$

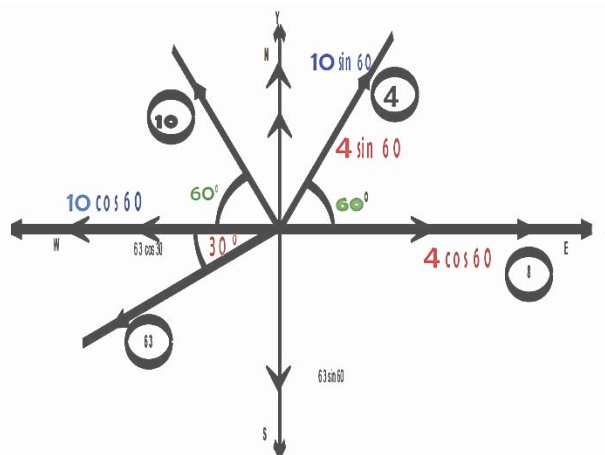
and

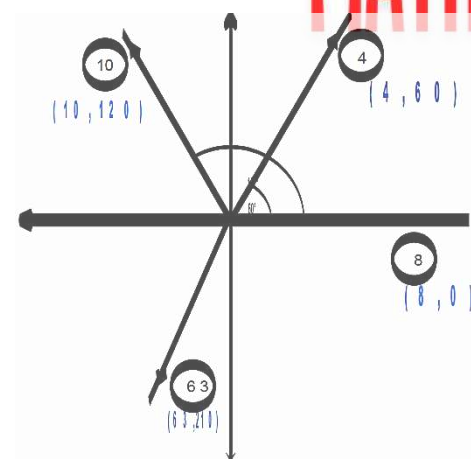
$$\tan \theta = \frac{Y}{X}$$



where θ is the polar angle of R .

1) Four coplanar forces act at a point, the 1st of magnitude 8N and acts towards east, the 2nd of magnitude 4N and acts in direction 60° north of east, the 3rd of magnitude 10N and acts in direction 60° north of west, and the 4th of magnitude $6\sqrt{3}$ and acts in direction 30° south of west, find the magnitude and direction of their resultant.





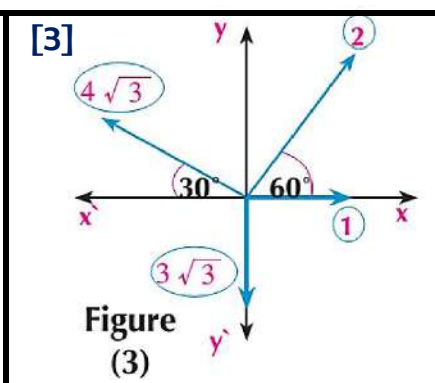
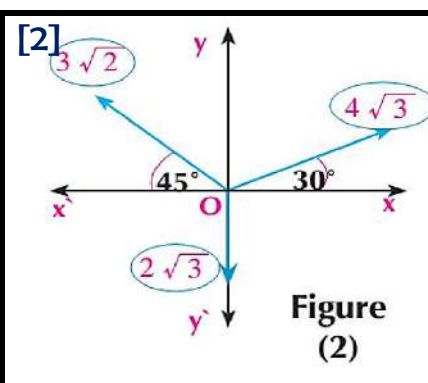
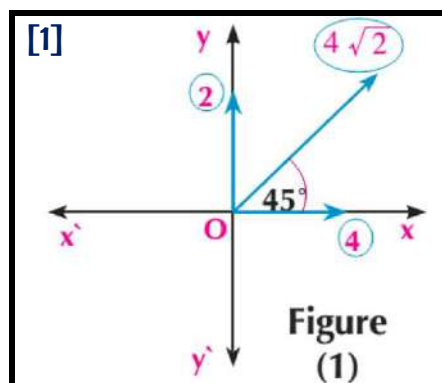
Complete each of the following:

[1] If the forces $\vec{F}_1 = 2\vec{i}$, $\vec{F}_2 = \vec{i} - 2\vec{j}$, $\vec{F}_3 = 6\vec{j}$ then: the magnitude of the resultant of the forces = and its direction

[2] If the forces $\vec{F}_1 = 2\vec{i} - 2\vec{j}$, $\vec{F}_2 = 4\vec{i} - 8\vec{j}$, $\vec{R} = 2a\vec{i} - 3b\vec{j}$ then $a = \dots\dots\dots$, $b = \dots\dots\dots$

[3] If the forces $\vec{F}_1 = 3\vec{i} - 2\vec{j}$, $\vec{F}_2 = a\vec{i} - \vec{j}$, $\vec{F}_3 = 4\vec{i} - b\vec{j}$, $\vec{R} = 6\vec{i} - 4\vec{j}$ then $a = \dots\dots\dots$, $b = \dots\dots\dots$

Find the magnitude and the direction of resultant of the forces shown in each of the following figures:



[4]

Isosceles triangle

Figure (4)

[5]

Rectangle it's dimensions are 6cm , 8cm

Figure (5)

[6]

Regular hexagon

Figure (6)

Answer the following questions:

① The forces 3 , 6 , $9\sqrt{3}$ and 12 kg.wt act on a particle and the measure of the angle between the first and the second is 60° , between the second and the third is 90° and between the third and the fourth is 150° . Find the magnitude and the direction of resultant of these forces.

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② Three forces of magnitudes 10 , 20 , 30 newton act at a particle. The first acts towards the east and the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° South of the west. Find the magnitude and the direction of resultant of these forces.

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③ Four forces of magnitudes 10 , 20 , $30\sqrt{3}$ and 40 gm.wt act on a particle, the first acts in the east direction and the second acts in the direction 60° north of the east and the third acts in the direction 30° north of the west and the fourth acts in the direction making an angle of 60° South of the east. Find the magnitude and direction of resultant of these forces.

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④ ABC is an equilateral triangle, M is the point of intersection of its medians. The forces of magnitudes 15 , 20 , 25 newton act on a particle in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$. Find the magnitude and the direction of the resultant of these

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⑤ ABCD is a square of side length 12 cm , $H \in \overline{BC}$ so $BH = 5$ cm. Forces of magnitudes 2 , 13 , $4\sqrt{2}$ and 9 gm.wt act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$, $\overrightarrow{AD}$ respectively. Find the magnitude of the resultant of these forces.

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⑥ If $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ are three coplanar forces meeting at a point and their resultant $\vec{R} = (10\sqrt{2}, 135^\circ)$, then find the values of a and b.

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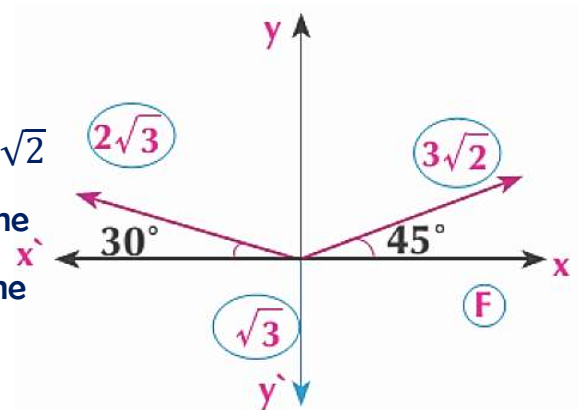
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⑦ In the opposite figure:

If the magnitude of the resultant of the forces equals $3\sqrt{2}$ Newton, then find the value of F and the measure of the angle between the line of action of the resultant and the first force.



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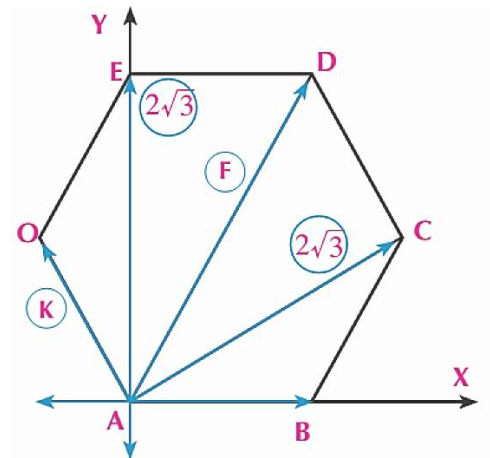
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⑧ In the opposite figure:

If the magnitude of the resultant of the forces equals 20 Kg.wt and acts in the direction of $\overrightarrow{AD}$ Find the values of F and K.



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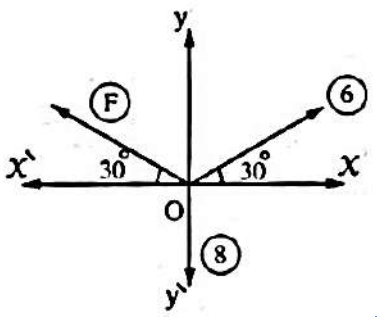
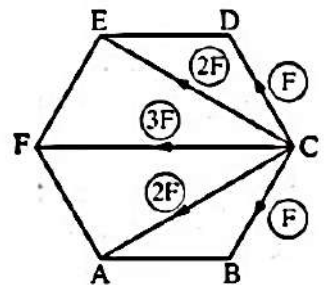
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Choose the correct answer:

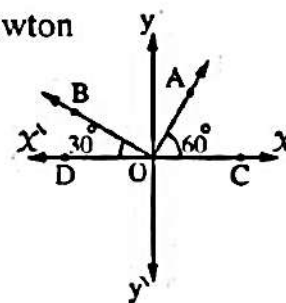
1)	If $\vec{F}_1 = \hat{i} - \hat{j}$, $\vec{F}_2 = 2\hat{i} - 4\hat{j}$, $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b = \dots\dots\dots$ (a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12	
2)	 If $\vec{F}_1 = 3\hat{i} - 2\hat{j}$, $\vec{F}_2 = a\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - b\hat{j}$, $\vec{R} = 6\hat{i} - 4\hat{j}$, then $(a, b) = \dots\dots\dots$ (a) (1, -1) (b) (-1, 1) (c) (-1, -1) (d) (1, 1)	
3)	If $\vec{F}_1 = 3\hat{i} + 2\hat{j}$, $\vec{F}_2 = a\hat{i} + 7\hat{j}$, $\vec{F}_3 = -12\hat{i} + b\hat{j}$ are three coplanar forces meeting at a point and the resultant $\vec{R} = (6\sqrt{2}, \frac{3}{4}\pi)$, then $a - b = \dots\dots\dots$ (a) -3 (b) 3 (c) zero (d) 6	
4)	Three coplanar forces $\vec{F}_1 = 6\hat{i} + 7\hat{j}$, $\vec{F}_2 = a\hat{i} - 9\hat{j}$, $\vec{F}_3 = 5\hat{i} + b\hat{j}$ act at a particle and they are in equilibrium , then $a + 2b = \dots\dots\dots$ (a) -9 (b) 5 (c) 7 (d) -7	
5)	If the resultant of the forces in the given figure acts in direction of y-axis , then $F = \dots\dots\dots$ force unit. (a) 2 (b) 6 (c) 8 (d) 14	
6)	The resultant of the forces in the opposite figure acts in direction of (a) $\vec{CD}$ (b) $\vec{CE}$ (c) $\vec{CF}$ (d) $\vec{CA}$	

7)

In the opposite figure :

The magnitude of four coplanar forces are 1, 2, $4\sqrt{3}$, $3\sqrt{3}$ newtonact at point O in the direction of $\overrightarrow{OX}$, $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OY}$, $m(\angle AOC) = 60^\circ$, $m(\angle BOD) = 30^\circ$,

then the magnitude and the direction of the resultant of the forces is

(a) (4, 180°)(b) (4, 0°)(c) (3, 0°)(d) (5, 90°)

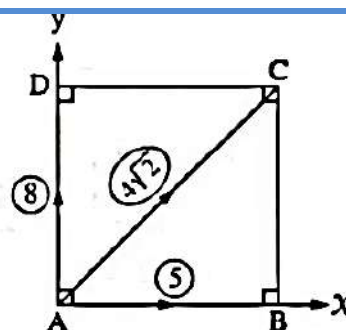
8)

In the opposite figure :

ABCD is a square, the forces of magnitudes

5, 8, $4\sqrt{2}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AC}$ respectively

, then the polar form of the resultant is

(a) (5, 54°)(b) (15, 60°)(c) (15, $53^\circ 8'$)(d) (13, 90°)

9)

In the opposite figure :

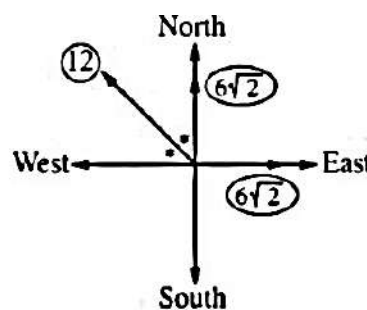
The direction of the resultant of the forces is

(a) South.

(b) East.

(c) West.

(d) North.



10)

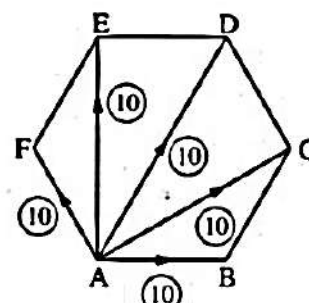
In the opposite figure :

Five equal forces each of magnitude 10 newton act

at one vertex of a regular hexagon and in direction of the other vertices of the hexagon, then the magnitude of the resultant of these forces = newton.

(a) 50

(b) 20

(c) $30\sqrt{3}$ (d) $20 + 10\sqrt{3}$ 

11)

In the opposite figure :

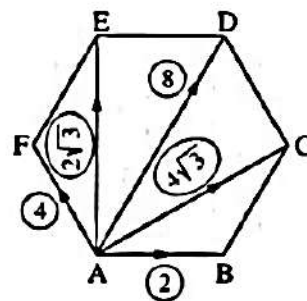
ABCDEF is a regular hexagon , forces of magnitudes $2, 4\sqrt{3}, 8, 2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}$ and $\overline{AF}$ respectively.

First : The magnitude of their resultant = kg.wt.

- (a) $14 + 6\sqrt{3}$ (b) 20
(c) $20\sqrt{3}$ (d) $20 + \sqrt{3}$

Second : The direction of the resultant inclined by an angle of measure with $\overline{AB}$

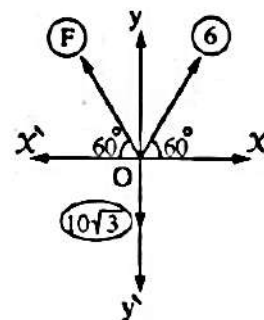
- (a) 30° (b) 45° (c) 60° (d) 90°



12)

If the resultant of the forces represented in the opposite figure acts in X-axis , then $F = \dots\dots\dots$ newton.

- (a) 10 (b) 14
(c) 18 (d) 6

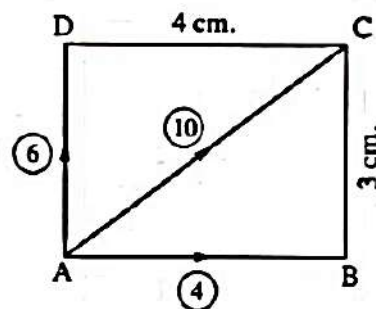


13)

In the opposite figure :

ABCD is a rectangle $AB = 4$ cm. , $BC = 3$ cm. forces 4 N , 10 , 6 N acts along $\overline{AB}, \overline{AC}, \overline{AD}$ respectively. The resultant of these forces makes with $\overline{AB}$ an angle of measure

- (a) 45° (b) 60°
(c) 30° (d) $\sin^{-1}\left(\frac{3}{5}\right)$



Equilibrium of a rigid body under the effect of coplanar forces meeting at a point

Main Points

- If a set of coplanar forces meeting at a point are completely represented by a side of a closed polygon, taken in the same cyclic order, then the forces are **equilibrium**.
- A set of coplanar forces meeting at a point is to be in **equilibrium**, if
 - 1) The sum of the algebraic components in the direction $\overrightarrow{OX} = \text{zero}$
 - 2) The sum of the algebraic components in the direction $\overrightarrow{OY} = \text{zero}$.
- **A body is in equilibrium under the effect of only two forces, means :**
The two forces are equal in magnitude, opposite in direction and their line of action are on the same straight line.
- **Transfer the point of action of the force:** if a force acts on a rigid body ,then it's possible to transfer its point of action to another point in the body on the same line of action of the force without any change of its effect on the body.
- **The equilibrium of a body under the effect of three forces:**
If a set of coplanar forces meeting at a point are represented by the sides of triangle taken in the same cyclic order, then these forces are equilibrium

1) If the three forces $\vec{F}_1 = 2\vec{i} + 5\vec{j}$, $\vec{F}_2 = \vec{i} - 8\vec{j}$, $\vec{F}_3 = m\vec{i} + n\vec{j}$, meeting at a point and in equilibrium, find the value of "m" and "n".

The system is in equilibrium

$$\therefore \vec{R} = 0 \quad \therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\therefore (2 + 1 + m) \vec{i} + (5 - 8 + n) \vec{j} = 0 \quad \therefore (m + 3) \vec{i} + (n - 3) \vec{j} = 0\vec{i} + 0\vec{j}$$

$$\therefore m + 3 = 0 \quad \therefore \underline{m = -3} \quad \text{and} \quad \therefore n - 3 = 0 \quad \therefore \underline{n = 3}$$

2) Three coplanar forces 8, 10, and 12 newtons act at a point, if the forces are in equilibrium, find the measure of the angle between the last two forces.

The forces are in equilibrium

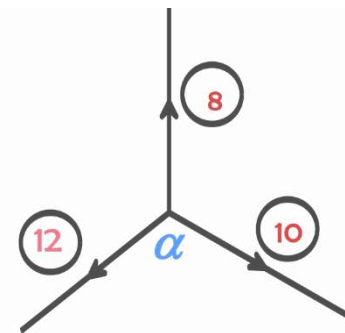
Then this resultant = 8

$$\therefore R^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha$$

$$\therefore (8)^2 = (10)^2 + (12)^2 + 2 \times 10 \times 12 \times \cos \alpha$$

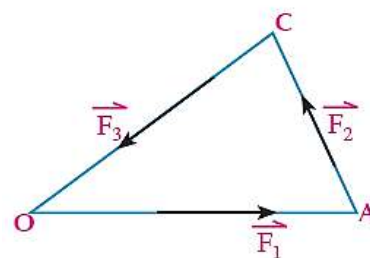
$$\therefore 240 \cos \alpha = -180 \quad \cos \alpha = -\frac{3}{4}$$

$$\therefore m(\angle \alpha) = 138^\circ 35'$$



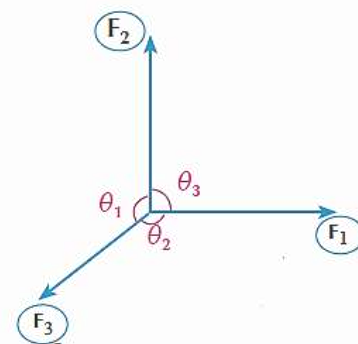
- **Triangle of forces rule:** If three forces acting at a point are in **equilibrium** and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order, then the lengths of the sides of the triangle are proportional to the magnitudes of the corresponding forces.

$$\frac{F_1}{OA} = \frac{F_2}{AC} = \frac{F_3}{CO}$$



- **Lami's rule:** If three forces meeting at a point and acting up on a particle are in **equilibrium**, then the magnitude of each force is proportional to the sine of the angle between the two other forces.

$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$



Types of questions

- (1) Body and two strings
- (2) Body, string and horizontal force
- (3) Body string and perpendicular force
- (4) Smooth inclined plane
- (5) Sphere
- (6) Rod

3) A weight 100 gm.wt is suspended by two strings of lengths 30 cm., and 40 cm., from two points on the same horizontal line with 50 cm a part. find the tension in each string.

From lami's rule

$$\therefore \frac{100}{\sin 90^\circ} = \frac{T_1}{\sin(180-\theta_1)} = \frac{T_2}{\sin(180-\theta_2)}$$

$$\therefore \frac{100}{1} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

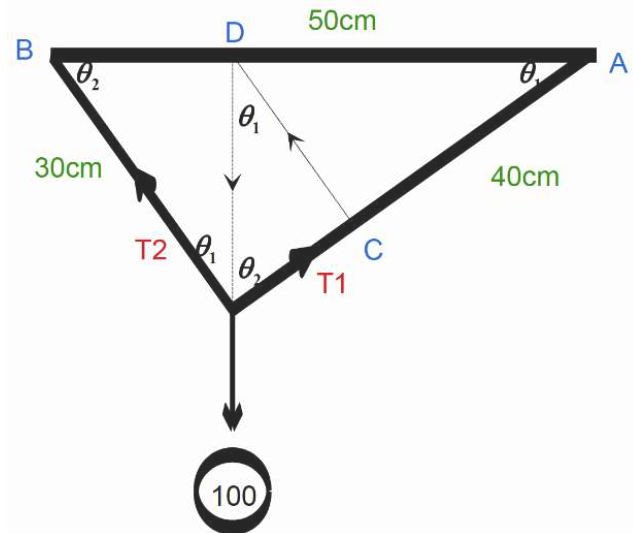
$$\sin \theta_1 = \frac{30}{50} = \frac{3}{5}$$

$$\sin \theta_2 = \frac{40}{50} = \frac{4}{5}$$

$$\therefore \frac{100}{\sin 90^\circ} = \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}}$$

$$\therefore T_1 = \frac{3}{5} \times 100 = 60 \text{ gm.wt}$$

$$\therefore T_2 = \frac{4}{5} \times 100 = 80 \text{ gm.wt}$$



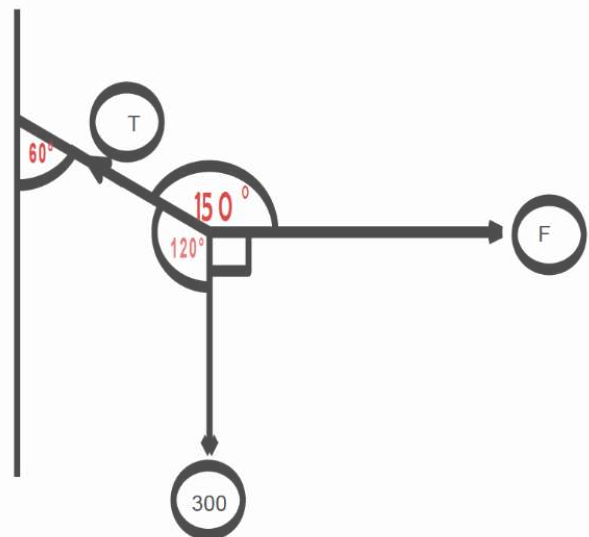
4) A body of weight 300 gm.wt is tied to an end of a string and the other end of the string is fixed at a vertical wall. If the body is pulled by a horizontal force until the string makes an angle of measure 60° with the wall in the state of equilibrium, Find the magnitude of the force and the tension in the string.

From lami's rule

$$\therefore \frac{F}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{300}{\sin 150^\circ}$$

$$\therefore T = \frac{300 \times \sin 90^\circ}{\sin 150^\circ} = 600 \text{ gm.wt}$$

$$\therefore F = \frac{300 \times \sin 120^\circ}{\sin 150^\circ} = 300\sqrt{3} \text{ gm.wt}$$



5) A body of weight 10 kg.wt is tied to an end of a string of length 170 cm., the other end of the string is fixed at a vertical wall, the body is pulled away of the wall by a horizontal force until it becomes 150 cm. apart from the wall in the state of equilibrium, Find the magnitude of F and T

In $\triangle OAB$ is right at A

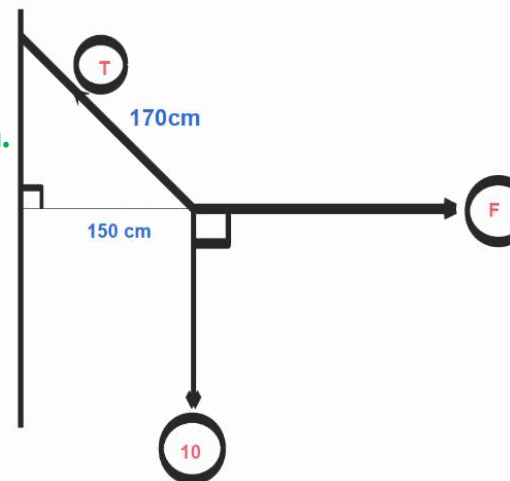
$$\therefore (OA)^2 = (OB)^2 - (AB)^2 = (170)^2 - (150)^2 \quad \therefore OA = 80 \text{ cm.}$$

$\triangle OAB$ is the triangle of forces

$$\therefore \frac{10}{80} = \frac{T}{170} = \frac{F}{150}$$

$$\therefore T = \frac{10 \times 170}{80} = \frac{85}{4} \text{ gm.wt}$$

$$\therefore F = \frac{10 \times 150}{80} = \frac{75}{4} \text{ gm.wt}$$



6) A body of mass 6 kg.wt is placed on smooth inclined plane 30° with the horizontal, is supported by force, Find this force and the reaction of the plane in each of the following cases.

first: the force is horizontal

second: the force is inclined by 30° to the plane

first: the force is horizontal

By using Lami's rule:

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 90^\circ} = \frac{6}{\sin 120^\circ}$$

$$\therefore F = \frac{6 \times \sin 150^\circ}{\sin 120^\circ} = 2\sqrt{3} \text{ gm.wt}$$

$$\therefore r = \frac{6 \times \sin 90^\circ}{\sin 120^\circ} = 4\sqrt{3} \text{ gm.wt}$$

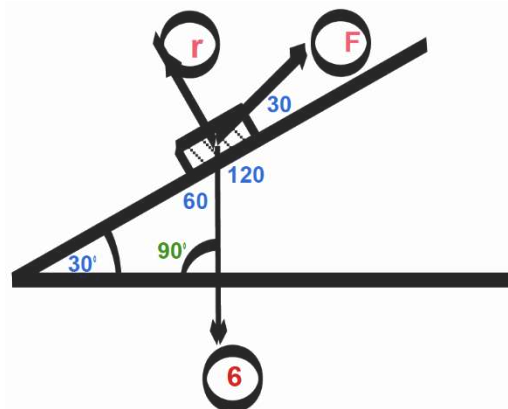
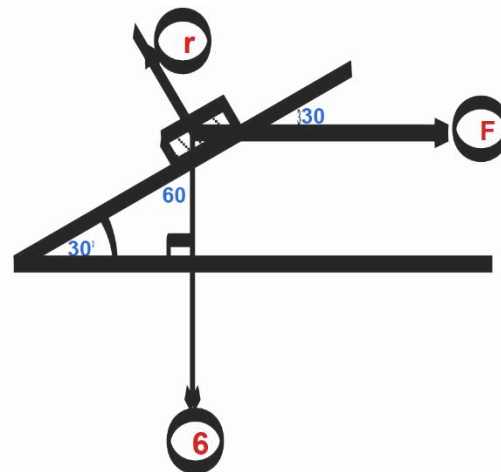
second: the force is inclined by 30° to the plane

By using Lami's rule:

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{6}{\sin 60^\circ}$$

$$\therefore F = \frac{6 \times \sin 150^\circ}{\sin 60^\circ} = 2\sqrt{3} \text{ gm.wt}$$

$$\therefore r = \frac{6 \times \sin 150^\circ}{\sin 60^\circ} = 2\sqrt{3} \text{ gm.wt.}$$



8) A body of weight "w" newtons is suspended by two strings, the first inclines by angle of measure θ to the vertical passing over a smooth pulley and carries at its other end a body of weight 12 newtons, the other string form with the vertical 30° , passing over a smooth pulley and carries at its other end a body of weight 8 newtons. Find θ and F

By using Lami's rule:

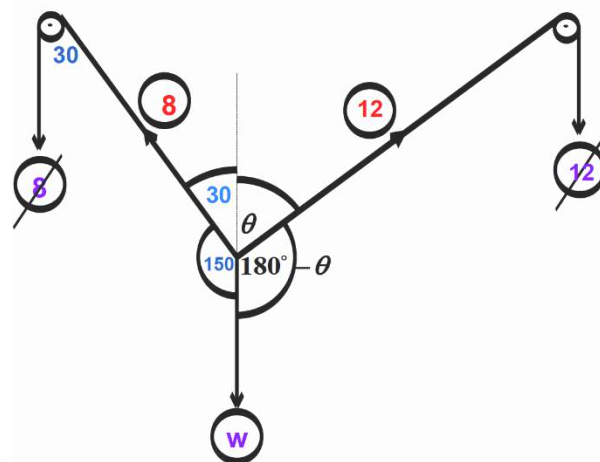
$$\therefore \frac{W}{\sin(30^\circ + \theta)} = \frac{12}{\sin 150^\circ} = \frac{8}{\sin(180^\circ + \theta)}$$

$$\therefore \sin \theta = \frac{8 \times \sin 150^\circ}{12} = \frac{1}{3}$$

$$\therefore m(\angle \theta) = 19^\circ 28'$$

$$\therefore \frac{W}{\sin(30^\circ + 19^\circ 28')} = \frac{12}{\sin 150^\circ}$$

$$\therefore W = \frac{12 \times \sin 49^\circ 28'}{\sin 150^\circ} \approx 18 \text{ newtons}$$



9) A body weight 400 gm.wt, is suspended by a string at a point A from a point B on the string, another string is attached and pulled horizontally by second string BC passing over a smooth fixed pulley and carries at its other end a body 300 gm.wt, find the inclination of AB with vertical and the tension in each of the two strings AB , BC

By using triangle of forces:

ΔADB is the triangle of forces

$$\therefore \frac{T_1}{BD} = \frac{T_2}{AB} = \frac{W}{AD}$$

$$\therefore \frac{300}{BD} = \frac{T_2}{AB} = \frac{400}{AD} \quad \therefore \frac{300}{BD} = \frac{400}{AD}$$

$$\therefore \frac{300}{400} = \frac{BD}{AD} = \frac{3}{4} \quad \therefore \tan A = \frac{3}{4}$$

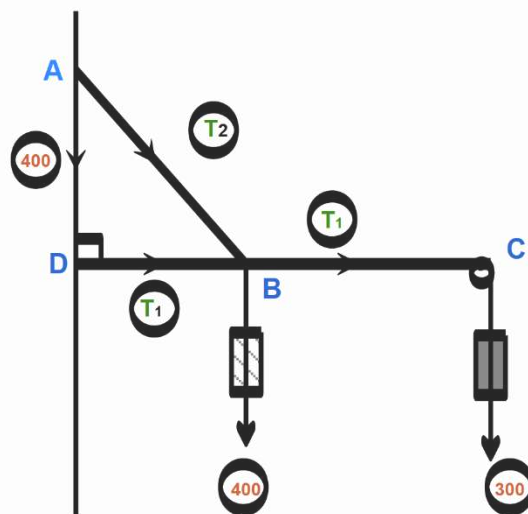
$$\therefore m(\angle \theta) = 36^\circ 52'$$

$$\therefore \text{angle } \theta \text{ inclination} = 36^\circ 52'$$

By using Lami's rule

$$\therefore \frac{300}{\sin 36^\circ 52'} = \frac{T_2}{\sin 90^\circ}$$

$$\therefore T_2 = \frac{300 \times \sin 90^\circ}{\sin 36^\circ 52'} = 500 \text{ gm.wt.}$$



Choose the correct answer:

1)	If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the of the included angle between the other two forces. (a) cosine (b) sine (c) tangent (d) cotangent	
2)	If a body is kept in equilibrium under action of several forces , then the least number of forces could cause equilibrium equals (a) 1 (b) 2 (c) 3 (d) 4	
3)	If $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of $\vec{F}_1$ and $\vec{F}_2$ = (a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero	
4)	Three equal forces in magnitude meeting at a point and they are in equilibrium , then the measure of the angle between each two forces = (a) 60° (b) 90° (c) 120° (d) 150°	
5)	If $\vec{F}$ is in equilibrium with two perpendicular forces of magnitudes 8 newton and 15 newton , then F = newton. (a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$	
6)	If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then F = newton. (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 6	
7)	The force which is in equilibrium with two perpendicular forces F , F newton makes with one of the two forces an angle of measure $^\circ$ (a) 90 (b) 120 (c) 135 (d) 150	

8)

Which of the following sets of forces could be in equilibrium ?

- ① 8 newton , 8 newton , 8 newton.
 ② 8 newton , 8 newton , 16 newton.
 ③ 8 newton , 8 newton , 20 newton.
 (a) ① only. (b) ② only. (c) ① , ② (d) ② , ③

9)

Three coplanar forces are in equilibrium act at a particle , the measure of the angle between the first two forces is 60° , and between the second and third forces is 150° then the ratio between forces is

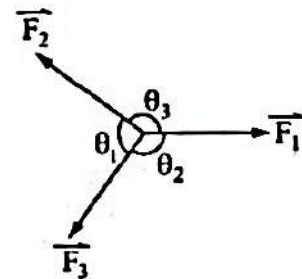
- (a) $1 : 1 : \sqrt{3}$ (b) $1 : 2 : \sqrt{3}$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $\sqrt{3} : \sqrt{3} : 1$

10)

If a body is in equilibrium under action of three forces as shown in the figure.

Which of the following statements is true ?

- ① $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \text{zero}$ ② $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{\text{zero}}$
 ③ $\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$



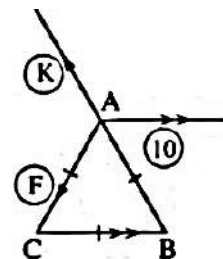
- (a) ① , ② only. (b) ② , ③ only.
 (c) ③ only. (d) ① , ② , ③

11)

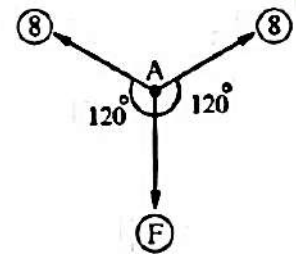
In the opposite figure :

If the three coplanar forces meeting at a point and in equilibrium , then $F = \dots\dots\dots$ N.

- (a) 10 (b) 12 (c) 20 (d) 13



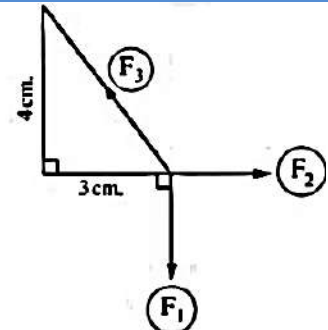
- 12) Particle A is balanced under action of three forces as shown in the opposite figure where $\vec{F}$ is balanced with two forces the magnitude of each is 8 newton and makes an angle of measure 120° with each of them, then $F = \dots\dots\dots$ newton.



- (a) zero (b) 8
(c) 16 (d) $8 \sin 120^\circ$

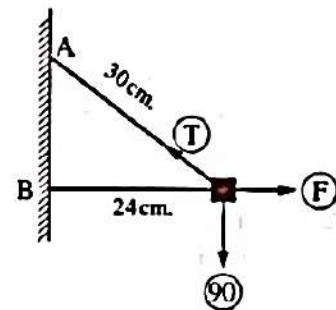
- 13) In the opposite figure :

A body is in equilibrium under action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order, then $F_1 : F_2 : F_3 = \dots\dots\dots$



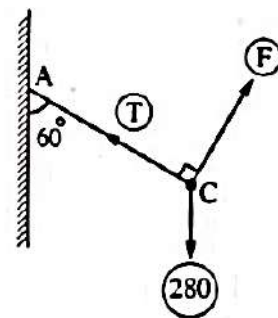
- (a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

- 14) In the opposite figure :
A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long. The body is pulled by a horizontal force. It comes to equilibrium when it is 24 cm. apart from the wall $\overline{AB}$ then $T - F = \dots\dots\dots$ gm.wt.



- (a) 150 (b) 120
(c) 50 (d) 30

- 15) In the opposite figure :
A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$



- (a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

16)

In the opposite figure :

The weight of a body is 20 kg.wt , the body is in

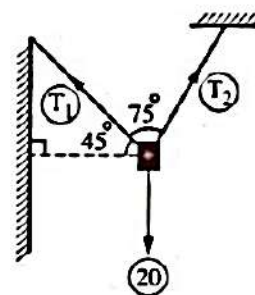
equilibrium then $\frac{T_1}{T_2} = \dots\dots\dots$

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{3}$

(d) $\frac{\sqrt{3}}{2}$



17)

In the opposite figure :

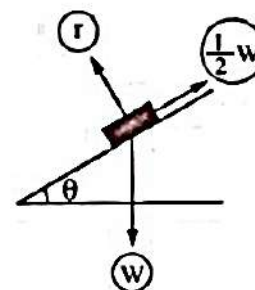
If the body is in equilibrium under action of forces shown , then $m(\angle \theta) = \dots\dots\dots$

(a) 30°

(b) 60°

(c) 45°

(d) 15°



18)

In the opposite figure :

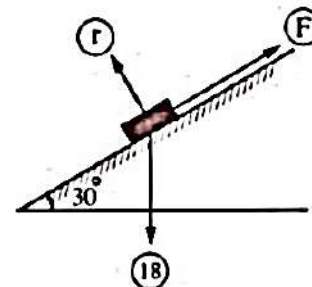
A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° , it is kept in equilibrium by a force of magnitude F newton in the direction of the plane upward , then $F + R = \dots\dots\dots$ newton.

(a) $6\sqrt{3}$

(b) $9\sqrt{3}$

(c) $18\sqrt{3}$

(d) $9 + 9\sqrt{3}$



19)

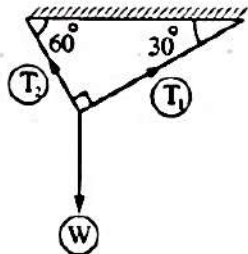
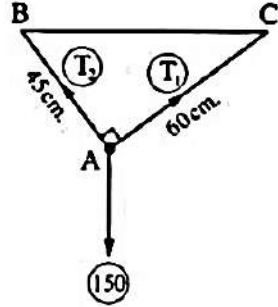
The weight of a body is 6 kg.wt. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium by a horizontal force , then the magnitude of this horizontal force = $\dots\dots\dots$ kg.wt.

(a) $\sqrt{3}$

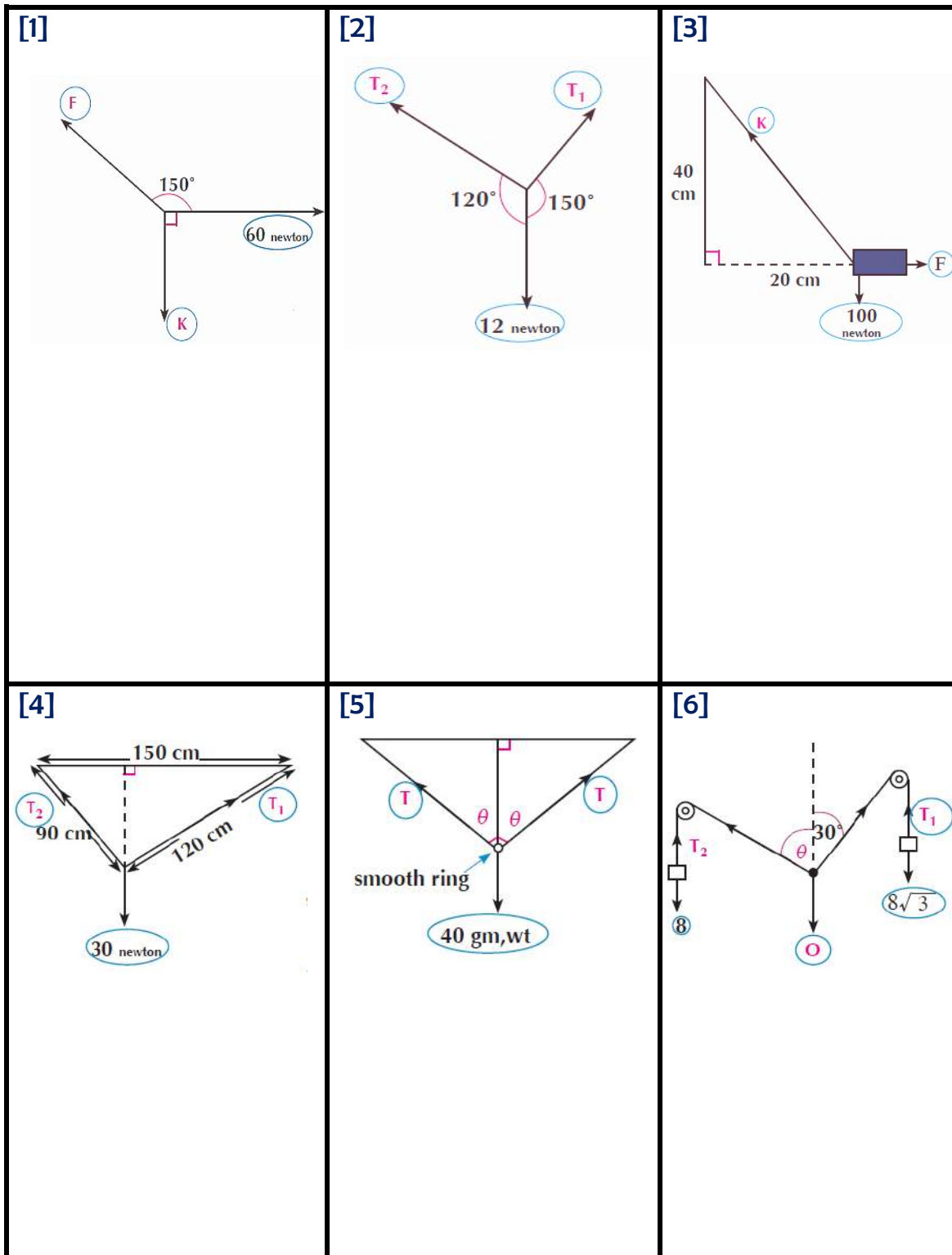
(b) $2\sqrt{3}$

(c) $4\sqrt{3}$

(d) 6

20)	In the opposite figure : A body of weight (W) is hanged by two strings. The two strings inclined to the horizontal as shown in the figure , then $T_1 = \dots\dots\dots$ (a) $\frac{1}{3} W$ (b) $\frac{1}{2} W$ (c) $\frac{\sqrt{3}}{3} W$ (d) $\frac{\sqrt{3}}{2} W$	
21)	In the opposite figure : A body of weight 150 gm.wt. is in equilibrium by suspending it by two perpendicular strings of lengths 60 cm. and 45 cm. , and the other two ends C and B are on a horizontal line , then : $T_2 - T_1 = \dots\dots\dots$ gm.wt. (a) 120 (b) 90 (c) 60 (d) 30	
22)	A body of weight 28 kg.wt. is suspended by two perpendicular strings , if the measure of the angle between one strings and the line of the weight is 120° , then the magnitude of the tension of this strings equals $\dots\dots\dots$ kg.wt. (a) 14 (b) 28 (c) $14\sqrt{3}$ (d) $28\sqrt{3}$	

Each figure from the following figures represents a set of coplanar equilibrium force meeting at a point. Find the value of the unknown (Force or angle):



Answer the following questions

[1] A weight of magnitude 60 gm.wt is suspended from one end of a string of length 28 cm. The other end is fixed at a point in the ceiling of the room, A force acts on the body so that the body becomes in equilibrium when it is about 14 cm vertically down the ceiling, If the force is in equilibrium position when it is normal to the string. Find the magnitude of each of the force and the tension in the string.

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[2] A weight of magnitude 200 gm.wt is hanged (suspended) by two strings of lengths 60 cm, 80 cm from two points on one horizontal line. The distance between them is 100 cm. Find the magnitude of the tension in each of the two strings.

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[3] A particle of weight 200 gm.wt is hanged (suspended) by two light strings.

One of the them inclines to the vertical with an angle of measure θ and the other string inclines to the vertical with an angle of measure 30° . If the magnitude of the tension in the first string equals 100 gm.wt, find θ and magnitude of the tension of the second string.

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[4] A body of weight 800 gm.wt is placed on a smooth plane inclined to the horizontal by angle of measure θ so that $\sin \theta = 0.6$. The body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

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[5] A body of weight (W) newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and the body is kept in equilibrium by the effect of force of magnitude 36 newton acts in direction of the line of the greatest slope upwards. Find the magnitude of the weight of the body and the magnitude of the reaction of the plane.

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Follow: The Equilibrium

1) From a point on the surface of a homogenous sphere, a light string is tied whose other end is attached to a point in a vertical smooth wall. if the sphere is stable on the wall, find the tension in the string and the reaction of the wall if the weight of the sphere is W newtons and acts at its center and the string inclined to the vertical by 30°

In this problem we find that the sphere is in equilibrium state under the action of three forces:

(1) The weight (W) act at M

(2) The reaction of the wall (R)

the wall is smooth, then the reaction is perpendicular to the plane, therefore, it passes through the center of the sphere M

(3) The tension in the string (T)

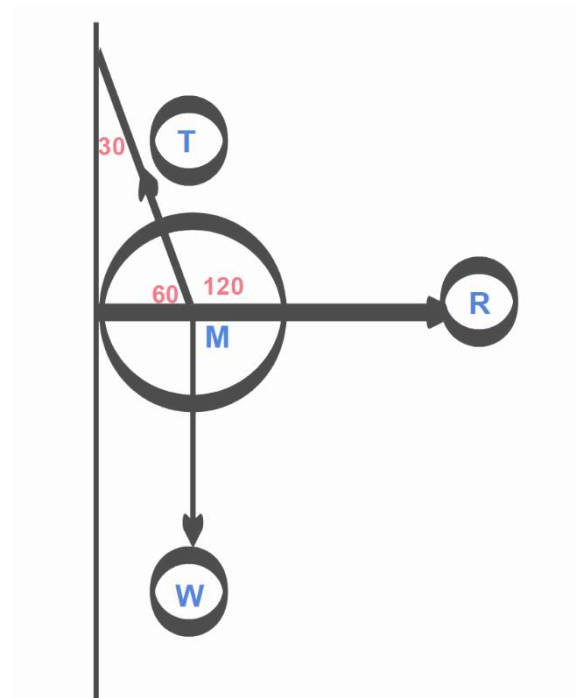
Since the line of the action of the weight and the reaction of the wall passes through the center M , then the line of action of the third force (the tension) must be pass through the point M

; By using Lami's Rule:

$$\therefore \frac{W}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{R}{\sin 150^\circ}$$

$$\therefore T = \frac{W \times \sin 90^\circ}{\sin 120^\circ} = \frac{2\sqrt{3}}{3} W \text{ Newtons}$$

$$\therefore R = \frac{W \times \sin 150^\circ}{\sin 120^\circ} = \frac{\sqrt{3}}{3} W \text{ Newtons}$$



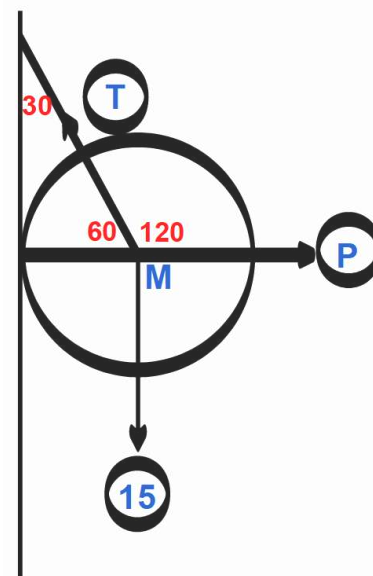
2) A smooth sphere of weight 15 newtons is on a smooth vertical wall and suspended by a light string from a point on its surface the other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere, if the length of the string equals the radius of the sphere, find the pressure on the wall and the tension in the string

By using Lami's Rule:

$$\therefore \frac{15}{\sin 120^\circ} = \frac{P}{\sin 150^\circ} = \frac{T}{\sin 90^\circ}$$

$$\therefore P = \frac{15 \times \sin 150^\circ}{\sin 120^\circ} = 5\sqrt{3} \text{ Newtons}$$

$$\therefore T = \frac{15 \times \sin 90^\circ}{\sin 120^\circ} = 10\sqrt{3} \text{ Newtons}$$



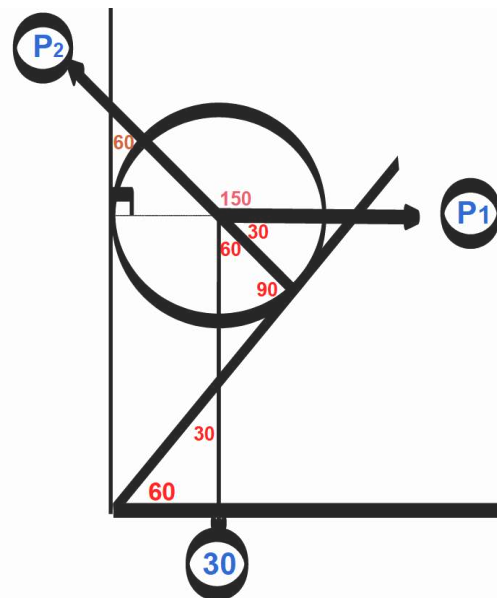
3) A smooth iron sphere of weight 30 newtons rests between a vertical smooth wall and an inclined smooth plane inclined by 60° with the horizontal, find the pressure on each of the wall and the plane

By using Lami's Rule:

$$\therefore \frac{P_1}{\sin 120^\circ} = \frac{P_2}{\sin 90^\circ} = \frac{30}{\sin 150^\circ}$$

$$\therefore P_1 = \frac{30 \times \sin 120^\circ}{\sin 150^\circ} = 30\sqrt{3} \text{ Newtons}$$

$$\therefore P_2 = \frac{30 \times \sin 90^\circ}{\sin 150^\circ} = 60 \text{ Newtons}$$

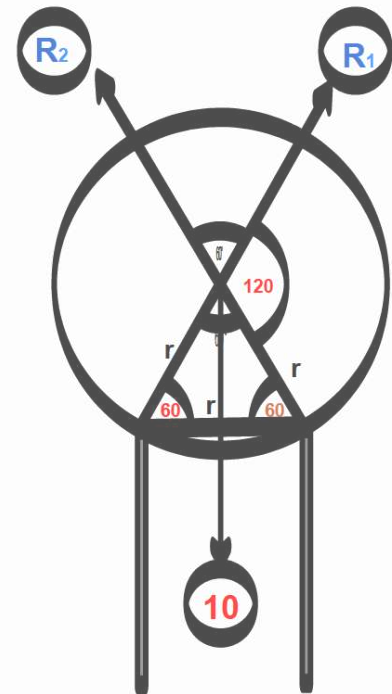


4) A sphere rests between two parallel rods lie in the same horizontal plane, the distance between them equals the radius of the sphere find the pressure on each rod given that the weight of the sphere is 10 newtons.

By using Lami's Rule:

$$\therefore \frac{R_1}{\sin 150^\circ} = \frac{R_2}{\sin 150^\circ} = \frac{10}{\sin 90^\circ}$$

$$\therefore R_1 = R_2 = \frac{10 \times \sin 150^\circ}{\sin 60^\circ} = \frac{10\sqrt{3}}{3} \text{ Newtons}$$



5) A uniform rod of 1 metre long and 30 newtons weight is suspended from both its ends by two strings their other ends are fixed at one point in the ceiling. if the two strings are perp. one of them is 60 cm., find the tension in each string when it is suspended freely and in equilibrium.

By using triangle of forces:

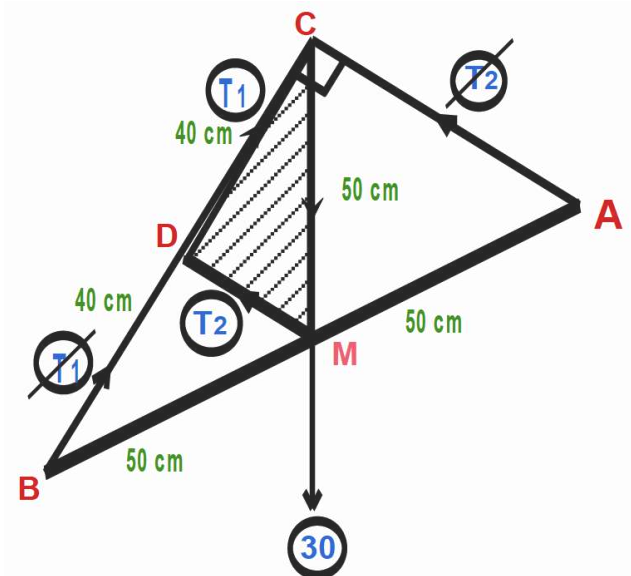
ΔDMC is the triangle of forces

$$\therefore \frac{30}{CM} = \frac{T_1}{DC} = \frac{T_2}{MD}$$

$$\therefore \frac{30}{50} = \frac{T_1}{40} = \frac{T_2}{30}$$

$$\therefore T_1 = \frac{40 \times 30}{50} = 24 \text{ Newtons}$$

$$\therefore T_2 = \frac{30 \times 30}{50} = 18 \text{ Newtons}$$



6) AB is a uniform rod of length 120 cm., and weight 300 gm.wt its end A is attached to a hinge fixed at a vertical wall, the other end B is attached to a string BC of length 150 cm., this string is fixed at a point C on the wall and lies vertically above A, the rod is in equilibrium when it is in horizontal position, find the tension in the string and the reaction of the hinge at A.

ΔABC is right at A $\therefore AC = \sqrt{150^2 - 120^2} = 90 \text{ cm.}$

$\therefore D$ is midpoint of AB, E is midpoint of BC

$\therefore AE$ is a median drawn from the vertex of right angle

$$\therefore AE = \frac{1}{2} BC = 75 \text{ cm}$$

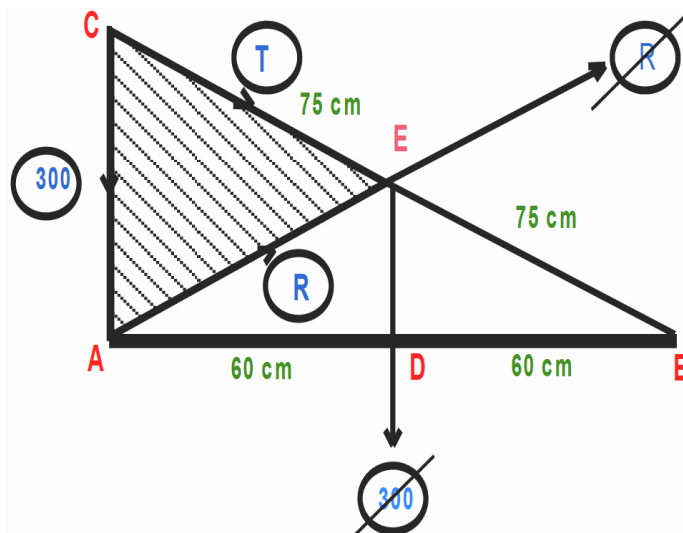
By using triangle of forces:

ΔAEC is the triangle of forces

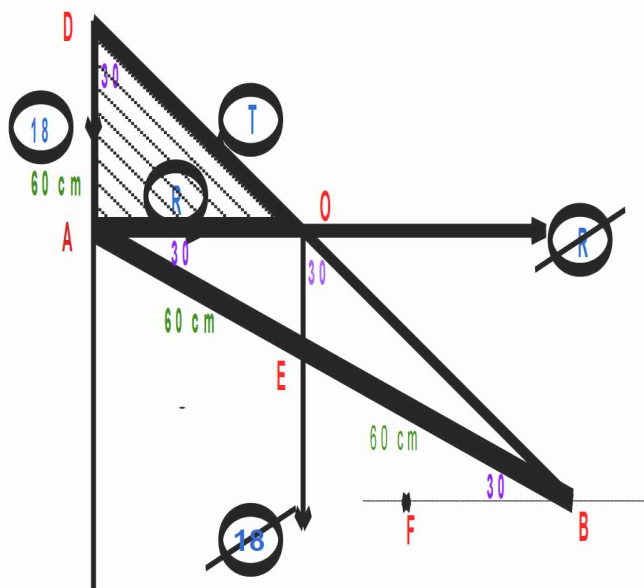
$$\therefore \frac{300}{CA} = \frac{R}{AE} = \frac{T}{CE}$$

$$\therefore \frac{300}{90} = \frac{R}{75} = \frac{T}{75}$$

$$\therefore R = T = \frac{75 \times 300}{90} = 250 \text{ gm.wt}$$

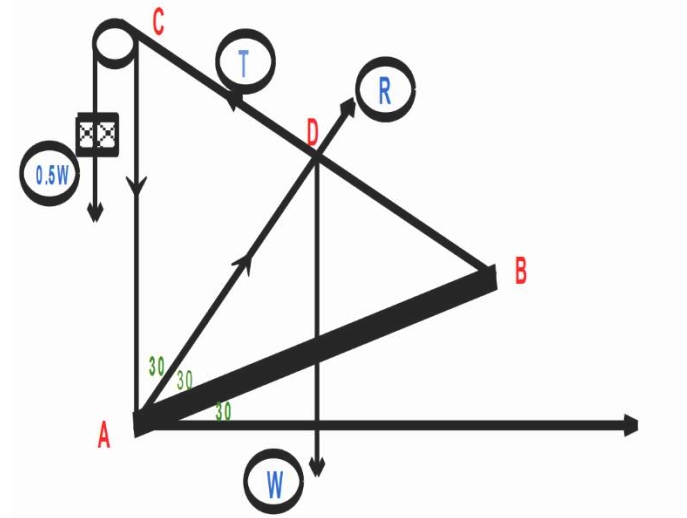


7) AB is a uniform rod of length 120 cm., and weight 18 km.wt it rotates about a hinge at its end A and its end B is pulled by a light string whose other ends is tied to a point D above A, the rod in equilibrium when it inclined to the horizontal at 30° , if the reaction of the hinge is horizontal. Find the tension of the string and the reaction of the rod in the wall



8) AB is a uniform rod hinged at A, the other end B is tied to a string pass over a smooth pulley C exactly above A and attached to a weight equals half the weight of the rod. Find the angle of inclination of the rod to the horizontal in state of equilibrium given that

$$AC = AB$$



EXERCISES

[1] A smooth sphere of Wight 200 gm.wt and of radius length 30 cm. rest against a smooth vertical wall. It is suspended at a point of its surface by means of a string of length 20 cm. ,and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. find the tension in the string and the reaction of the wall.

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[2] A smooth sphere of Wight $10\sqrt{3}$ gm.wt. rests against a smooth vertical wall. It is suspended at a point of its surface by means of a string and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. If the string makes with the vertically an angle of measure 30° . find the tension in the string and the reaction of the wall.

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[3] A smooth metal sphere of weight 3 Newton at rest (stable) between a smooth vertical wall and a smooth plane inclined to the vertical wall with angle of measure 30° . Find the pressure on each of the vertical wall and the inclined plane.

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[4] A rod of length 50 cm and weight 20 newton was hanged (suspended) from its terminals with two strings such that the two ends are fixed in one point. If the length of the two strings are 30 cm , 40 cm respectively. Find the tension in each of the two strings.

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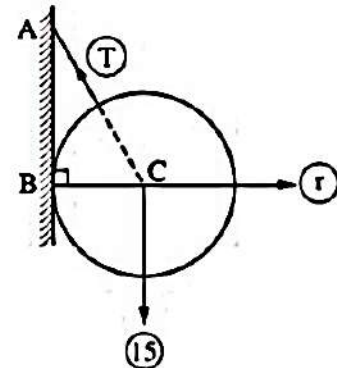
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Choose the correct answer:

1)

In the opposite figure :

A solid uniform sphere of weight 15 gm.wt. and radius length 10 cm. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then $(r, T) = \dots\dots\dots$



(a) $(4\sqrt{3}, 8\sqrt{3})$

(b) $(5\sqrt{3}, 10\sqrt{3})$

(c) $(5, 10)$

(d) $(5\sqrt{3}, 8\sqrt{3})$

2)

In the opposite figure :

If the sphere is in equilibrium

, then $T - r = \dots\dots\dots$ newton

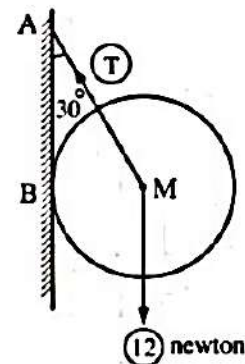
(Where r is the magnitude of the wall reaction on the sphere)

(a) $8\sqrt{3}$

(b) $4\sqrt{3}$

(c) 4

(d) 8



3)

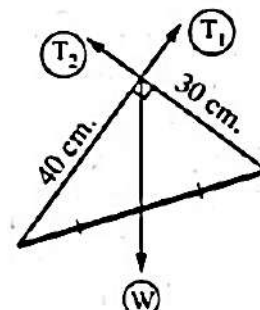
In the opposite figure : $T_1 : T_2 : W = \dots\dots\dots$

(a) 5 : 3 : 4

(b) 8 : 5 : 4

(c) 4 : 3 : 5

(d) 5 : 4 : 3



4)

In the opposite figure :

$\overline{AB}$ is a uniform rod with length 40 cm. and weight 30 newton is connected to a hinge at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and C where C is located on the wall just above A , $AC = 40$ cm.

First : The reaction of the hinge $r = \dots\dots\dots$ newton.

(a) 30

(b) 20

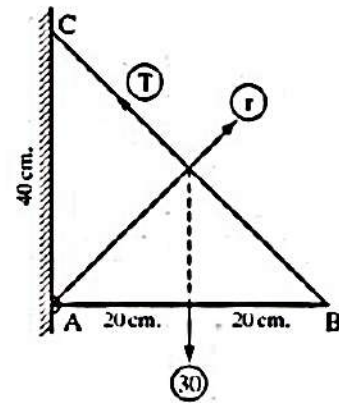
(c) $40\sqrt{2}$ (d) $15\sqrt{2}$

Second : The tension of the string $T = \dots\dots\dots$ newton.

(a) $15\sqrt{2}$

(b) 30

(c) 20

(d) $40\sqrt{2}$ 

كيفية طباعة صفحات معينة من ملف معين مثلا ازاي نطبع الصفحات من صفحة 4 الى صفحة 9

